Forming groups

Students using grouping strategies…

understand that numbers are capable of being regrouped in flexible ways. These students no longer need to rely on counting by ones but can apply a range of grouping strategies to solve addition and subtraction problems.

When dealing with multiplication and division problems, students at this level are able to form equal groups. Rather than emphasising individual items or numbers, the students develop an increasingly sophisticated idea of "composites". The students are able to focus on a group of items and learn to treat the group as a single item.

Array structures assist students to "see" composite groups and using this structure aids students to quickly find the total using the process of multiplication.

When students are able to group units and treat them as composites, they then need to move from needing the items present, to being able to visualise and use multiple counting strategies to determine the total of the groups.

As students begin to use composite units, it is common to see them use a combination of counting strategies to determine the total.

Just as students counting strategies are limited by their knowledge of the sequence of number words, so too their early multiplication and division strategies are often limited by their knowledge of the sequence of multiples. When a student's knowledge of a sequence of multiples is exhausted, they will switch back to counting by ones.

As students work with larger numbers, the use of grouping begins to draw upon the special role played by "tens" in our number system.

Place value concepts, as with multiplication and division processes, make use of the composite nature of groups, but in this case the groups are units of, tens, hundreds, thousands and so on. Addition, subtraction, multiplication and division all make use of place value for both mental and written computation.

The notion of a composite unit is also important in the development of measurement concepts. Knowledge of the size of units and being able to repeatedly use a unit are critical to the process of measuring. Students need to see how individual units can be combined into composite units, for example a row of tiles or a layer of bricks. These composite units can then be used in calculations such as in determining area multiplication.

In spatial understanding, individual units can also be combined into composite units. For example, combining two squares to make a rectangle, repeating and rotating patterns and tiling activities.

Index of activities

Memory domino

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Encourage students to share their strategies.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

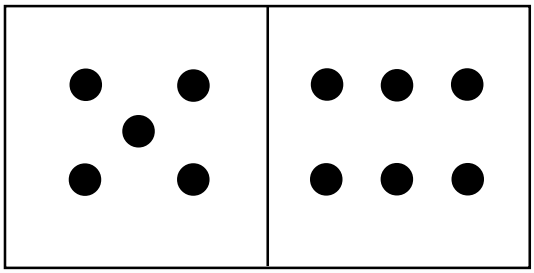
WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

How?

Organise a set of dominoes representing number combinations to 12 and a set of numeral cards in the range 0–12. Place the dominoes and the numeral cards face down on the floor. Have the students take turns to firstly flip over one domino. Encourage the student to immediately state the number of dots on each side of the domino and then the total. Then have the students flip over a numeral card to try and match the total on the domino. If the student successfully matches a pair they keep the domino and the numeral card.



Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Domino challenge

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

How?

Organise a set of dominoes so that all the tiles greater than five are placed face down on the floor. Quickly turn over a domino and then place it back on the floor face down. The first student to call out the total number of dots on the domino keeps the tile. If a student calls out an answer, another student may call "challenge" if he or she believes it is incorrect. The "challenger" may keep the tile if he or she states the correct number. If the challenger is incorrect then the tile is returned to the central pile or to the first caller if he or she was correct. Continue until all tiles have been claimed.

Variation

Use a set of "double 9" dominoes. Leave the tile turned over after the student has called out the total. If both the "caller" and the "challenger" are incorrect the tile is then turned over.

Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Ten-frame flash

Where are they now?

Students are able to recognise a small number of items instantly, without the need to count each item.

Where to next?

Students are able to recognise larger groups of items when arranged in spatial patterns.

Make a variety of ten-frames including some displaying dots that are not next to each other, for example, one or two spaces between dots.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: conceptual

BLM

Ten-frame, page 148

How?

Prepare flash cards showing ten-frames containing various dot patterns. Hold a ten-frame horizontally and briefly flash the ten-frame. Have the students record how many dots are in the top row, how many dots are in the bottom row and how many dots altogether. This leads to a natural progression of recording a written algorithm, when the numbers are recorded vertically. Have the students share their responses.

Variation

Prepare two sets of ten-frame cards. Each card displaying a filled, or partially filled, ten-frame. Do not use scattered patterns. Flash a ten-frame from each set. Have the students determine how many dots are on each card and then determine how many dots there are altogether.

Why?

Activities that promote the development of instant recognition of spatial patterns can assist students to recall number facts.

Largest number wins

Where are they now?

Students can count forwards and backwards by 10s on and off the decade.

Where to next?

Students can count forwards and backwards by 100s off the 100 and on or off the decade to 1000.

Discuss with the students how they know which is the largest number. Lead discussions and follow-up activities to renaming units within numbers. For example, seeing that "302" is the same as 30 tens and 2 ones, assists consolidation of place value concepts.

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

Numeral identification: level 4

How?

Organise the students into small groups and provide them with an "operation die" (A cube marked with "+1", "-1", "+10", "-10", "+100", "-100".) Each player starts with a score of 500. The die is rolled and each player adds or subtracts the number rolled to his or her score. In turns, players then have four rolls of the "operation die". After each roll the player calculates and records his or her tally. The winner is the player with the largest number.

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when solving some two- and three-digit computations mentally.

Start with four

Where are they now?

Students can count forwards and backwards by tens on and off the decade.

Where to next?

Students can count forwards and backwards by 100 on or off the decade to 1000.

The "operation die" from *Largest number wins* could be used instead of the "operation cards".

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

Numeral identification: level 4

BLM

Start with four, page 236

How?

Organise the students into pairs or groups of three and provide them with a set of numeral cards 1–9 (make at least three of each number), a set of instruction cards "+1" "+10" "+100" "-1" "-10" "-100" (at least three of each) and a recording sheet each. Alternatively, use the operation die from *Largest number wins*. Ask the students to shuffle the numeral cards and deal out four cards to form a four-digit number. This will be the starting number for the first round. Each student records the starting number on his or her worksheet. The students then take turns to draw an instruction card and add it to, or subtract it from, the starting number and record the new tally on the worksheet. Play continues until all players have had four turns at drawing an "instruction card". The player with the largest number after four draws is the winner.

Variation

Instead of having a "winner", have the students record the final tally of each player as a group recording, by sequencing the numbers from lowest to highest.

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when mentally solving some addition and subtraction problems.

Bucket count on: 10s and 100s

Where are they now?

Students can count forwards and backwards by tens on and off the decade.

Where to next?

Students can count forwards and backwards by 100 on or off the decade to 1000.

Syllabus outcomes

NS2.1: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and 100s: level 3

How?

Drop a small collection of large disks or blocks (all of one colour) into a bucket or container one at a time. Tell the students that the colour of the discs, say red, represents a unit of ten. Ask the students to count aloud by tens as each disc is added. Choose a different coloured disc and tell the students that this colour, say blue, represents units of 100. Drop the discs into the bucket one at a time. Ask the students to continue counting by adding on 100 to the total as each disc is dropped. After adding in this fashion, return to adding discs representing "tens" to the total. Discs of another colour could be used to represent units of "one" and if appropriate, use discs to represent units of "1000".

Why?

Being able to count on and off the decade and off the hundred is a useful strategy for students to apply when mentally solving some addition and subtraction problems.

I have, I want, I need

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

A traditional number line is usually segmented and marked with numerals at regular intervals. The empty number line (without pre-written numerals or interval marks) provides the student with the control to place chosen numerals at any points along the line. The focus is therefore on how the problem was solved and can reflect the student's mathematical thinking without imposing a set procedure.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

CMIT reference

Building place value through grouping: level 2

Combining and partitioning: To 10

BLM

I have, I want, I need, page 237

How?

Provide each student with a worksheet and a set of numeral cards in the range 0–9. Have the students draw two cards from the pile and construct the lowest two-digit number possible from the combination. Record the numeral under the "I have" box. The student then reverses the numerals and records the new number under the "I want" box. Have the students determine the difference between the two numbers and use the empty number line to record their problem solving strategy. Record the difference in the "I need" box. Have the students share their strategies with the class. If the same number is drawn for both cards, have the students return one to the pile and redraw another card.

Variation

Use playing cards instead of numeral cards.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Model the use of the empty number line as a way of recording. Demonstrate strategies such as "building to the next decade", "jumping by multiples of ten" and "counting down to a number".

The empty number line

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

TP

Model the use of the empty number line as a way of recording. Demonstrate strategies such as "building to the next decade", "jumping by multiples of ten" and "counting down to the decade".

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

CMIT reference

Building place value through grouping: level 2

Combining and partitioning: To 10

How?

Demonstrate the use of the empty number line to students, showing how multiples of tens and single digits can be recorded and used to solve addition and subtraction problems.

Provide the students with two piles of numeral cards in the range 0–9. Have the students draw two cards to create a two-digit number. Ask the students to use an empty number line to record the way they found out how many more are needed to make 100.

Variation

Provide the students with two sets of cards. The students then draw four cards and make two, two-digit numbers. Have the students use the empty number line to record their methods of adding the two numbers. Encourage the students to build to the next decade and then count on by tens or groups of ten.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing formal algorithms.

Hundred chart challenge

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Discuss the mental calculations students use to solve problems and which strategy the students think was the best. For example, did they count-on by tens and ones and keep track of the total? Did they bridge to the nearest decade? Did they use the hundred chart to track counting on by tens and then add or subtract the ones?

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

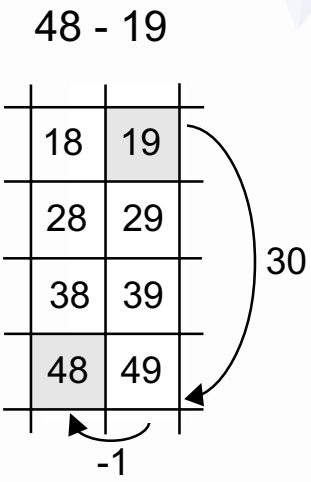
Building place value through grouping: level 2

Combining and partitioning

Recording symbols

How?

Prepare two packs of numerals cards in the range 0–9. Have the students select two cards from each pack to make two, two-digit numbers. Locate and mark the numbers on a hundred chart. Have the students determine the difference between the two numbers in the least number of moves. Encourage the students to explain how they determined the difference.



"Add thirty, go back one."

Why?

Problems involving regrouping of numbers both with and without material should be introduced to the students before teaching procedures for algorithms. Students need significant opportunities to focus on regrouping numbers to develop an understanding of place value.

WR

Record students thinking on the chalkboard as solution strategies are explained.

Addition challenge

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Model mental computation strategies such as combining numbers to make ten or adding on by tens and then ones.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

Building place value through grouping: level 2

Combining and partitioning

Recording symbols

How?

Organise the students into small groups and provide each group with a set of playing cards, using the "ace" (to represent number one) through to the "nine", and paper for recording. Divide the cards into two piles. Students take turns to draw two cards to make a two-digit number. Each student then draws another two cards to form a second two-digit number, adds the two numbers together and records the total. The aim is to be the player with a total of 100 or to have the largest total less than 100. A player with a total greater than 100 automatically loses. Instruct the students to keep a tally of their wins and the first to score ten wins and is the "grand champion".

Variation

Use a hundred chart to assist mental calculations.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Use the students recording sheets to discuss and model strategies.

Friends to 100

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Demonstrate to the students how a subtraction problem may be solved using an addition process. This could be based on the "jump" method of adding tens and ones to the first number.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

CMIT reference

Building place value through grouping: level 2(a)

BLM

Friends to 100, page 238

How?

Organise the students into pairs and provide each pair with a copy of a hundred chart. The first student calls out a number between 1 and 99. The second student shades over the called number on the hundred chart and then counts on from the number by tens and ones to determine how many more are needed to make 100. After determining the difference, the student locates this number on the hundred chart and shades over the number. Discuss any patterns that the students notice. After practice, have the students determine the difference without the use of the hundred chart.

Variations

Record the number combinations, as they are determined.

Pose the problem as a subtraction, e.g. *How many would I have to take away from one hundred to land on the called number?*

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing formal algorithms.

Red or black

Where are they now?

Students are able to find the total of a pair of two-digit numbers by counting by tens and ones, with the use of materials.

Where to next?

Students use a range of counting strategies to find the total of two, two-digit numbers mentally.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

Building place value through grouping: level 2

Recording symbols

How?

Each group will need a pack of playing cards and paper to record on. Each player begins with a score of 20 points. Shuffle the cards and place them face down in a pile. Each student in the group takes turns to draw a card from the pile. Before doing so, he or she must say whether the card will be red or black. If the student correctly guesses the colour of the card, the number on the card is added to his or her score. If the student guesses incorrectly, the number on the card is subtracted. If the student guesses incorrectly, but hasn't enough points to take away the number on the card, he or she misses a turn. All cards have a score of their face value. Picture cards score as follows:

Jack = 11, Queen = 12, King = 13, Ace = 1

The winner is the first player to score 100.

Variations

Remove picture cards and play with cards Ace–10 only.

Students invent extra rules for playing. For example, include the "joker" and if turned up, the player reverses the score of an opponent.

Correct guesses are added to the score. Incorrect guesses miss a turn.

Change the winning score. For example, for a shorter game change the winning score to 50.

Highest score after five turns wins.

Why?

Students should have the opportunity to develop mental strategies for solving addition and subtraction problems and to record their thinking before introducing algorithmic procedures.

Have the students record their mental calculations for solving the two-digit additions and subtractions. Different ways of recording can be used as follow-up topics of discussion.

Double dice multi

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples without having to rely on perceiving the items.

Provide many opportunities for students to practise forwards and backwards counting of multiples.

Syllabus reference

WMS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

BLM

Double dice multi, page 239

How?

Provide the students with a baseboard, two dice, one die displaying numerals 1 to 6, the other displaying numerals 1 to 3 and 12 counters. Instruct the students to take turns to roll the dice and multiply the two numbers rolled. Model to the students how skip counting or repeated addition may be used to determine the answer. Once the answer has been determined, the student covers the corresponding numeral on the baseboard. If the number is already covered, the player misses a turn. Continue until all numerals on the baseboard have been covered.

Variations

Modify the dice to have both displaying numerals 1–6 or replace one of the dice with a ten-sided die displaying numerals 1–10. If varying the dice, the base-board will need to be modified.

Use three dice. Have the students roll the dice and choose two of the numbers rolled to multiply.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

Teddy target

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Encourage the students to use skip counting or repeated addition to determine the totals.

Point out to the students that the constant function on the calculator is using repeated addition.

Syllabus reference

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

How?

Draw a large target on the asphalt with chalk. Write the numerals 6, 5, 4 and 3, on the target, so that one numeral is on one segment of the target. Organise the students into teams and provide each team with a set of "teddy bags". "Teddy bags" are clear plastic bags containing 2, 3, 4, 5 or 6 plastic teddies. Have the students take turns to throw the "teddy bags" onto the target. The team calculates the score by multiplying the number of teddies in the "teddy bag" by the number indicated on the target segment. Have one of the team members record the score and another member check the calculation on a calculator. After each member has had a throw, the team adds the total. The team with the highest total wins.

Variations

Change the number of teddies in the "teddy bags."

Change the numerals on the target.

Use cloth beanbags marked with numerals to encourage students to recall multiplication facts.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

Have the students show their thinking when recording the totals. Use students' recordings to discuss and model strategies.

Collecting threes

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Syllabus outcomes

NS2.3: Uses mental and informal written strategies for multiplication and division

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3

Recording symbols

How?

Organise the students into small groups. Have the students take turns to roll three dice. Whenever a three or a six is rolled, the player records the number of threes and the total. Each player needs to keep a personal total. The first player to reach "30" is the winner.

Variation

Roll the three dice and use any combinations to form groups of three. For example if a three, five and four are thrown, the student may record four groups of three and a total of twelve.

Why?

Students need to move from being dependent on using groups of physical objects to being able to keep track of the groups when the material is not present.

People markers

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Syllabus outcomes

WMS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

Recording symbols

How?

Prepare ten dot-pattern cards for the multiple to be practised. For example, ten cards showing four dots on each. Distribute the dot pattern cards to the students and tell them they are going to represent the group on the card. Nominate a number of students, say six, to bring their dot pattern cards, face down, to the front of the room and form a line. Have the students use skip counting or repeated addition for the nominated multiple to determine the total as each person turns over his or her card in turn. Repeat the activity using various numbers of students in the line and record the number facts.

Variation

Use numeral cards instead of dot pattern cards.

Why?

Before practising multiplication tables by rote or other methods, students should have a good understanding of using composite units, without having to rely on visual representations of the units.

Students could record the multiples on a number line first and then record the multiplication fact.

Triples plus one

Where are they now?

Students are able to form equal groups and count perceived items by ones.

Where to next?

Students are able to count forwards or backwards using a pattern of multiples or repeated addition without having to rely on perceiving the items.

Model strategies of skip counting and repeated addition to the students and encourage them to use these strategies whilst completing the activity.

Syllabus outcomes

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

BLM

Triples plus one, page 240

How?

Prepare a baseboard for each pair of students and also provide them with a pile of counters and a numeral die. Have the students take turns to roll the die. The student then multiplies the number rolled by three, determines the answer, and then adds one more to the total. The student then covers a corresponding numeral on the baseboard with a counter. The first player to cover four numerals in a row, vertically, horizontally or diagonally wins.

Why?

Before practising multiplication tables by rote or other methods, students should have a good understanding of using composite units, without having to rely on visual representations of the units.

Graphing with symbols

Where are they now?

Students are able to form equal groups and find the total by counting by ones.

Where to next?

Students use skip counting or repeated addition to determine the total of the groups.

Students' strategy use in solving multiplication and division problems will be dependent upon their knowledge of multiple sequences.

Syllabus outcomes

DS2.1: Gathers and organises data, displays data using tables and graphs, and interprets the results

NS1.3: Uses a range of mental strategies and concrete materials for multiplication and division

WMS2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas.

CMIT reference

Building multiplication and division through equal grouping and counting: level 3 and 4

How?

Organise the students to investigate number problems where they have to find the total number and represent the quantity in symbols.

For example:

- The number of fingers in the room.  = 5 fingers
- The number of car doors in the car park.  = 2 doors
- The number of wheels on cars in the carpark  = 4 wheels
- The number of trees in the playground  = 10 trees

Have the students represent the information as a picture graph and have other students in the class interpret the graphs.

Why?

These activities allow the student to be removed one step from direct use of concrete materials and counting by ones. Here the symbols act as markers for the students to keep track of the groups as they count the multiples.