Developing Efficient Numeracy Strategies

Stage 2

NEW SOUTH WALES DEPARTMENT OF EDUCATION AND TRAINING

PROFESSIONAL SUPPORT AND CURRICULUM DIRECTORATE

STAGE 2

Developing efficient numeracy strategies

Acknowledgement

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Developing efficient numeracy strategies: Stage 2

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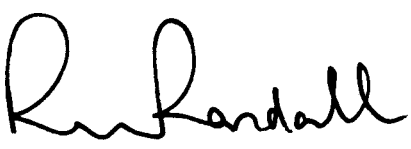
Foreword

Supporting the effective learning of mathematics relies on understanding what students know and need to learn, and providing opportunities for all students to learn with understanding. Learning mathematics with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future. Students' understanding of mathematical ideas can be built throughout their school years if they actively engage in tasks and experiences designed to deepen and connect their knowledge.

One of the central understandings that students develop in Stage 2 is the coordination of multi-units or groups of groups. Students' understanding of the base-ten number system is deepened as they come to understand its multiplicative structure. That is, 253 is seen as 2 x 100 plus 5 x 10 plus 3 x 1 as well as a collection of 253 individual objects. This type of multiplicative reasoning is further developed as students use a rectangular array as a geometric model of multiplication, and adapt this model to work out the area of shapes and the volume of solids.

Developing efficient numeracy strategies: Stage 2 also focuses on the role played by recordings in sharing thinking. Finding multiple ways to represent ideas lies at the heart of mathematics. We need to help students to learn how to talk about mathematics, to explain their answers, and to describe their strategies.

We work in exciting times in education. Never before has there been such widespread adoption of the findings of educational research operating in mathematics classrooms. I commend *Developing efficient numeracy strategies: Stage 2* as a resource to aid in the implementation of the new *Mathematics K–6* syllabus and to assist in strengthening the communities of mathematics learners in our schools.



Robert Randall
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About this book

Developing efficient numeracy strategies: Stage 2 is organised into three main sections based on increasingly efficient counting and grouping strategies. The three sections are: Counting by ones, Forming groups and Coordinating groups.

• Counting by ones

The activities within this section are intended to develop strategies students use beyond counting-based strategies. Within the *Count Me In Too* project's learning framework, these students would typically be identified as being at the *Counting on and back stage* of early arithmetical strategies.

• Forming groups

The activities within this section are intended for students who no longer rely on counting-by-ones strategies and are able to apply a range of grouping strategies to solve problems. Within the *Count Me In Too* project's learning framework, these students would typically be identified as being at the *Facile stage* of early arithmetical strategies.

• Coordinating groups

The idea of coordinating groups is fundamental to multiplication and division, place value, measurement and spatial concepts. The activities within this section are intended to develop students' use of collection-based strategies to solve problems through a focus on mental computation.

Assessing your students' strategies

An overview of the key strategies students use at each of these developmental stages is provided at the start of each section.

You will need to ascertain your students' current problem-solving strategies and counting skills in order to choose the most appropriate teaching activities.

Managing parts and wholes

Recognising and manipulating *parts* and *wholes* is basic to all mathematics development. Individual numbers, such as one, two and three, become parts of other numbers (2 + 3 = 5), individual shapes become parts of other shapes and parts of shapes such as angles become things of interest by themselves.

Students learn to do arithmetic by becoming adept at counting and developing methods to keep track of completed counts of multiples. As students learn to tie together number names and completed counts, they also learn to combine these *number facts*. As their skills grow, they tend to rely less and less on counting. Combining and partitioning become the basis of arithmetical operations.

Just as *counting* describes more than generating a sequence of words, *grouping* captures a range of procedures. Grouping procedures include the formation of groups as units, instant recognition of units, combining and separating groups within a unit and coordinating groups of groups. Managing parts and wholes in grouping aids the development of place value understanding.

Understanding the relationships between parts and wholes is important in all strands of mathematics. Within the Space strand, the focus and nature of students spatial understanding develops from shifting attention from the whole or global features of shapes and objects such as triangles or boxes, to the parts. The relationship between the parts and the wholes results in the recognition of the properties of shapes and solids.

In the Measurement strand, measuring deals with identifying a unit that can be used, without gaps or overlaps, to break the whole into identical parts. Length, area and volume require repeated applications of a standard unit to create the whole. The number of units used provides a way of quantifying space.

The notion of "units within units" is important for separating and combining numbers as well as for multiplication, division, fractions and measurement.

The focus of this book is supporting the development of students' strategies in counting and grouping of parts and wholes.

Place value

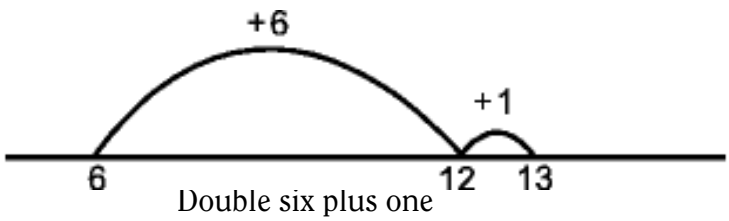
Interpreting number in terms of part–whole relationships makes it possible for students to think about a number as being made up of other numbers. This involves recognising that a number such as 19 can be partitioned into smaller groups of 10 and 9, as well as 11 and 8, 12 and 7 and so on. Breaking the whole into a range of parts can contribute to a richer understanding of place value that is needed to underpin the four operations with numbers.

As students work with larger numbers, grouping and re-grouping draws on the role played by *tens* in our number system. Grouping within place value involves the student understanding ten as a composite unit. That is, ten is composed of ten ones at the same time as being one unit called ten. The composite nature of ten allows it to be collected as multiples of ten as well as being re-formed by trading ten ones.

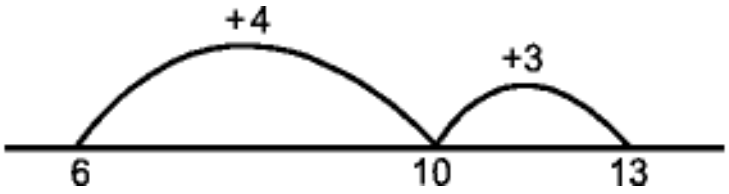
The mental methods that we use to answer questions such as 3 x 299 or 25 + 79 draw on a rich understanding of tens in numbers. For the second example, this knowledge of tens and hundreds leads us to recognise that 80 is 1 more than 79 or 25 + 75 = 100. This understanding of the flexible ways of using tens, hundreds and units with mental computation forms an essential foundation for formal algorithms.

Using recordings to share thinking

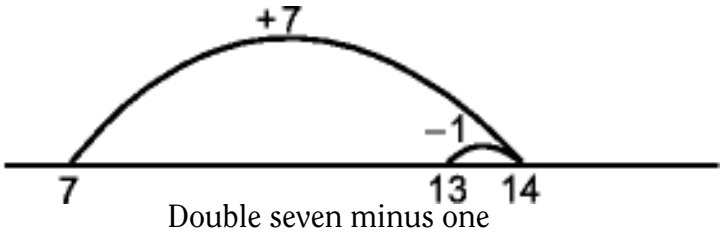
Students' explanations of their methods of working with numbers can also provide evidence of the increasing sophistication of strategies they use. Even asking students to explain how they think about finding the answer to a number combination such as 6 + 7, can provide a wealth of information. The diverse methods of constructing an answer to this question can make use of recordings as a way of sharing thinking. The use of the *empty number line* as a method of recording thinking enables recordings to support the sharing of thinking. It also aids in building a mental model to assist thinking.



Double six plus one



Bridging to ten



Double seven minus one

Purpose and direction

This book contains activities designed to build upon the students' current understandings and problem-solving strategies across the strands of *Number, Patterns and algebra, Data, Measurement* and *Space and geometry*.

The organisation of the material in this book emphasises both direction and purpose in the teaching of mathematics. It is based on the understanding that assessment of students' current knowledge is essential in planning appropriate teaching programs. This is sometimes referred to as assessment *for* learning.

The sections are sequenced to reflect development from counting by ones strategies, to grouping strategies, to the ability to coordinate groups of groups. Within each section the activities have been arranged to answer the fundamental questions of teaching mathematics:

What do my students currently know?

What do I want them to know?

How will I help them to know this?

How will I know when they know this?

How to use this book

The double-page layout of *Developing efficient numeracy strategies: Stage 2* organises four important aspects for planning effective teaching programs:

- identifying the students' current knowledge
- recognising the next step in the student's learning
- organising learning content to facilitate student progress
- identifying the purpose of the learning content and how it relates to the learning process.

These four aspects are addressed under the headings:

Where are they now?

(Identifying the students' current knowledge)

This component describes the types of approaches that students may use in attempting to solve problems.

Where to next?

(Recognising the next step in the students' learning)

This component provides direction for teachers in determining where students are heading in their learning.

How?

(Organising learning content)

This component outlines activities designed to assist students' mathematics development. The activities are suggested models for developing students' understanding and should be modified to suit the individual needs of the students and the available resources.

Why?

(Identifying the purpose of learning)

This component outlines the purpose of the activities.

Syllabus outcomes

Links have been identified between the activities and the 2002 *Mathematics Years K–6* syllabus outcomes.

Count Me In Too

Links have been identified between the activities and the *Count Me In Too* learning frameworks.

Short, practical assessment ideas are included at the end of each section. These ideas for assessing your students' development will enable you to determine if your students have progressed to the next stage of learning.

BLM

Black line masters (BLM) have been provided for many of the activities and are located at the end of each section.

Where a BLM has been provided for an activity, the title of the BLM and page number are shown.

Assessment tasks

Quick, practical assessment tasks are included at the end of each section. These tasks provide insight into students problem-solving strategies and will enable you to determine if students have progressed to the next stage of development.

Maths bites

Short, whole-class activities requiring little or no equipment are presented. These can be used to consolidate and practise skills and concepts, or as warm-up introductory activities to a lesson or as transition activities between lessons.

Other key features include written recordings and teaching points. The following icons identify these features:

Written recordings

Teaching directions and models for students' recording are linked to activities.

Teaching points

Ideas to help you organise the activities and further detail for teaching strategies are provided.

Icons are also used throughout the book to indicate whether the activities are appropriate for individuals, partners, small groups or whole class.


Individuals


Partners


Small group


Whole class