Counting by ones

Students using counting by ones strategies…

can often develop effective methods of solving problems. They may even become quite quick at using a count-by-one process. We have seen many students use elaborate finger strategies to assist their counting by ones, in particular with multiplication and division tasks. It is easy to assume these students are progressing satisfactorily, especially as this method of counting can give a correct answer.

While counting-by-ones is a satisfactory strategy for adding small numbers, it becomes a laborious process, often prone to miscalculations, when solving more complex problems involving larger numbers. These students need to develop efficient grouping strategies for solving problems.

The transition from counting-based strategies to collection-based methods is an important development. We need to assist students in developing a range of solution strategies beyond counting-by-ones.

The strategies described below are characteristic of students who have moved beyond a dominant use of non-count-by-ones strategies. These methods are typical of the *facile* counting stage.

Compensation for addition and subtraction. Seven plus three is the same as eight plus two. Sixteen take away nine is the same as seventeen take away ten.

Commuting for addition. Two and nine is the same as nine and two.

Using addition for subtraction. Eight and four is twelve, so twelve take away four is eight.

Using doubles. Nine and nine is eighteen, so eight and nine is seventeen.

Using a known fact. Four and three is seven, so twenty-four and three is twenty-seven.

Partitioning using five as a base. Four and three is the same as four and one and two.

Partitioning using ten as a base. Seven and six is the same as seven and three and three.

Using the tens-structure. Fifteen take away four is eleven because fifteen take away ten is five and four is one less than five.

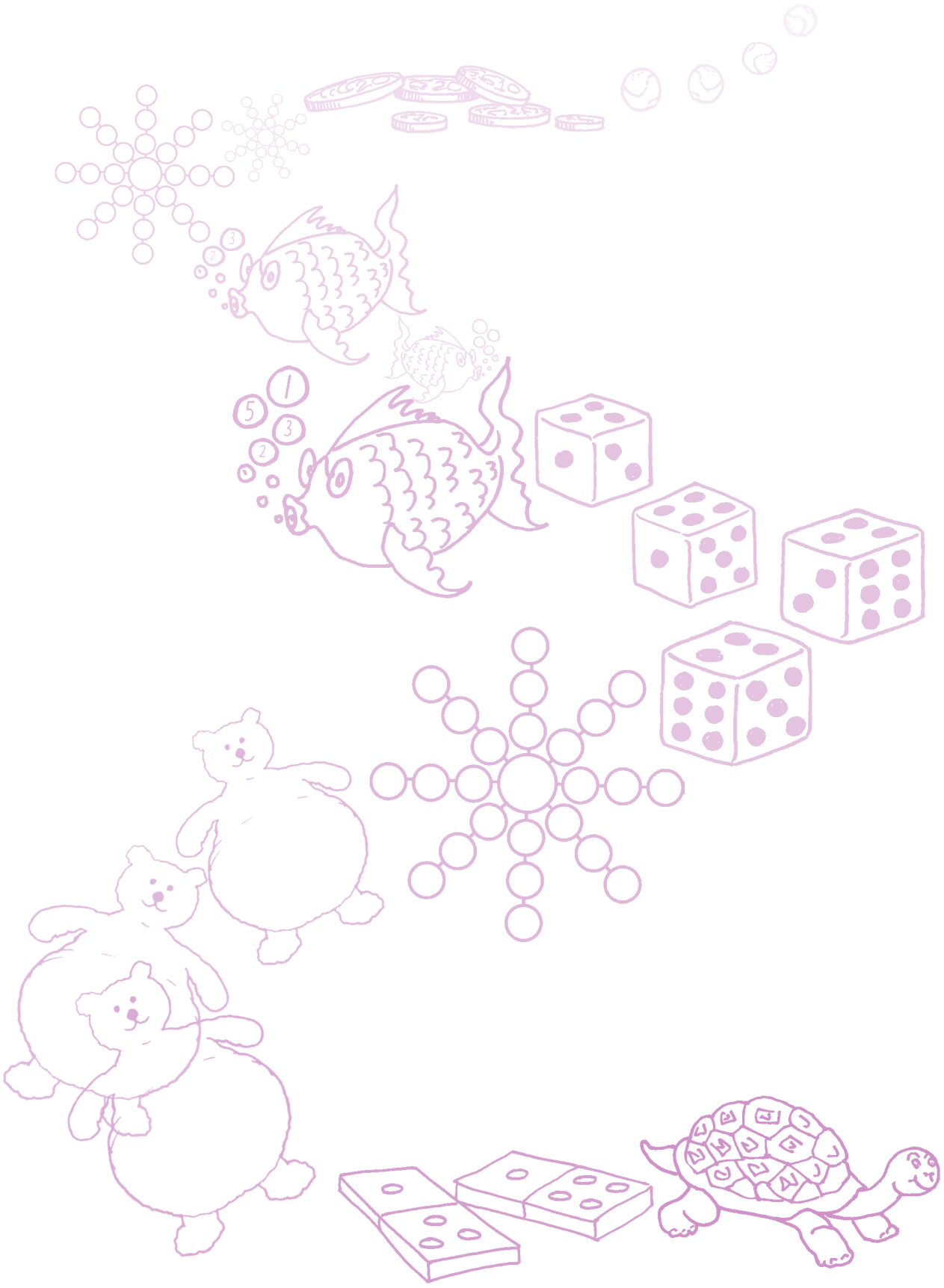
Understanding that numbers can be regrouped is essential if students are to move beyond the reliance on counting-by-ones. For example, when solving 64 – 18 seeing the number eighteen as eighteen ones is more limiting than also seeing that it can be flexibly regrouped, say into fourteen and four.

Combining and partitioning refers to the joining and separating of groups. Experiences with combining and separating groups can lead to the learning of number facts in a meaningful way.

The idea of groups within numbers lends itself to emphasising part–whole relationships, that is the student sees both the parts and the whole. This idea is also fundamental to place value, multiplication and measurement.

Part–whole relationships are also important to spatial understanding. In "Space" mathematics, part–whole relationships refer to how a shape is part of a larger shape. Students need to develop both visual imagery and appropriate language to describe shapes and objects as they analyse and mentally manipulate the parts as well as the "whole".

Index of activities



Domino adding pairs

Where are they now?

Students are able to add pairs of numbers to 20 by counting-on by ones.

Where to next?

Students use a range of non-count-by-one strategies including doubles, near-doubles and bridging to the next decade to solve addition problems.

TP Model making tens, using doubles or counting multiples to solve addition problems.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WM2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

Combining and partitioning: level 1

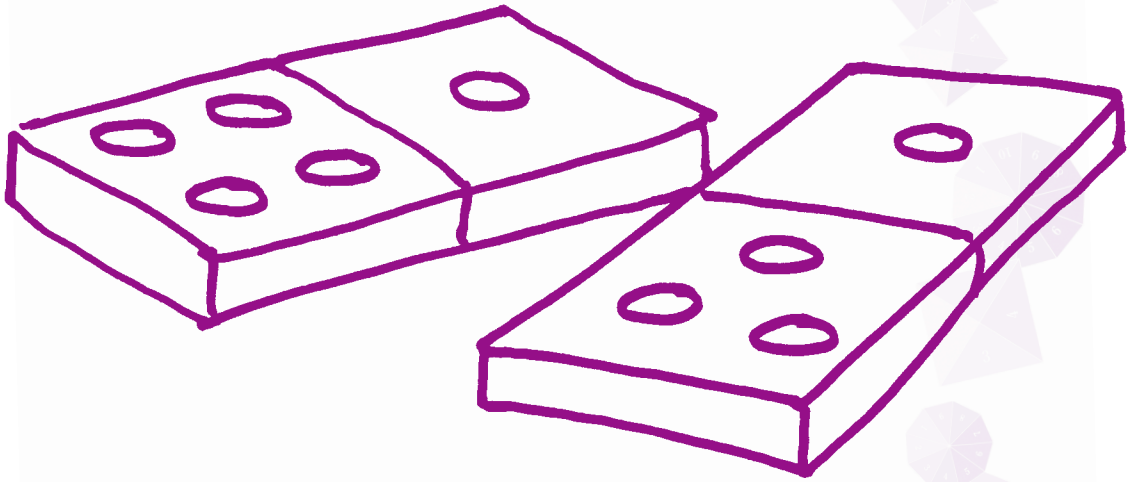
Subitising: level 1, 2

BLM

Domino adding pairs, page 136

How?

Provide each pair of students with copies of *Domino adding pairs* BLM and a set of dominoes. Remove the "double blank" domino tile. Arrange the dominoes face down. Have the students take turns to select a pair of dominoes and place them onto the worksheet. The students initially record each dot pattern as a numeral and then determine and record the total for each domino. The students then calculate the total of the pair of dominoes. Ensure the students discuss how they are completing the sum and record the procedure they used on the worksheet.



Why?

Students need to develop a variety of non-count-by-one strategies to assist them in solving addition and subtraction problems in an efficient way.

Domino friends

Where are they now?

Students use count-by-one strategies to solve addition problems.

Where to next?

Students use a range of non-count-by-one strategies including doubles, near-doubles and bridging to the next decade to solve addition problems.

TP

Have the students model non-count-by-one strategies to solve addition problems prior to commencing the activity.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WM2.3: Uses appropriate terminology to describe, and symbols to represent, mathematical ideas

NS2.5: Describes and compares chance events in social and experimental contexts

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

How?

A large space is required for this activity. Organise the class so that each student draws a domino tile from a bag without the rest of the class seeing the tile. Once each student has a domino tile, instruct them to move around the room and call out the numbers represented by the dot pattern until they find a "friend" whose numbers total to the same as their own tile. The "friends" can then continue calling out their number to see if they are able to form a group with the same total. After the activity ask the students to look around to see which total had the best chance of finding a friend. Discuss with the class why some students could not find partners and why there were more of some numbers.

Variation

Have the students find friends that together have a total of say, 12 or more.

Why?

Students need to develop a variety of non-count-by-one strategies to assist them in solving addition and subtraction problems in an efficient way.

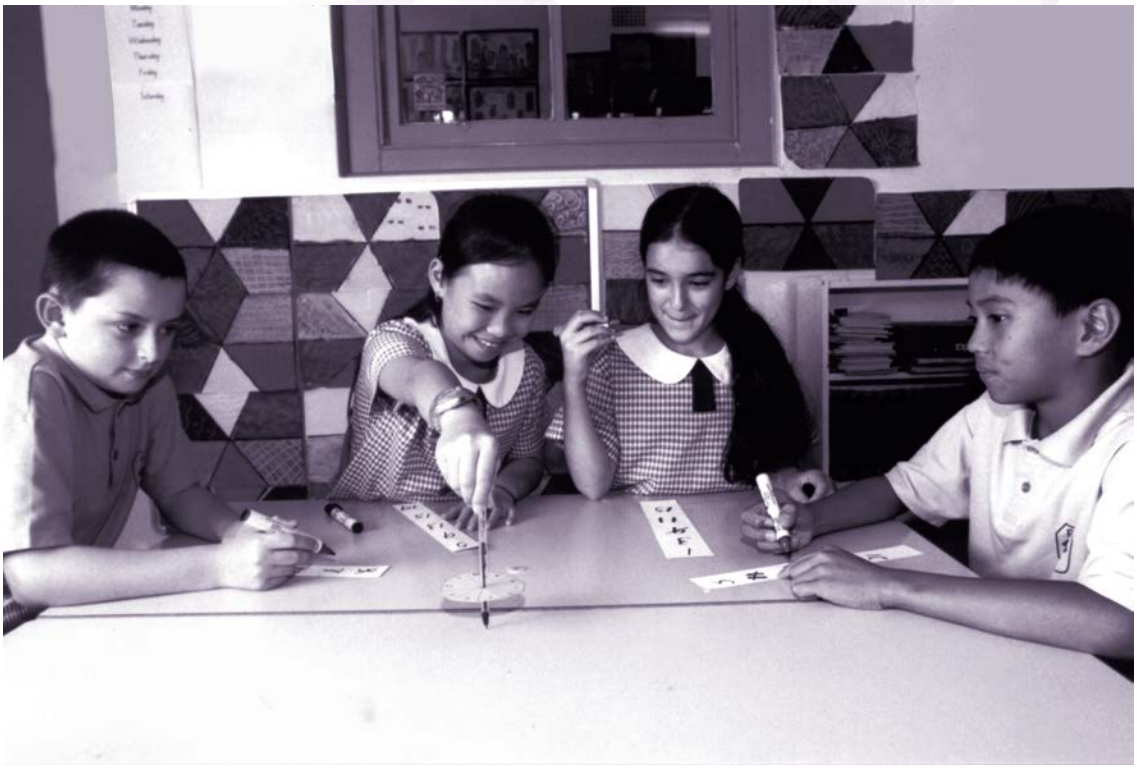
Spin, double and flip

Where are they now?

Students are able to solve addition and subtraction questions by counting by ones.

Where to next?

Students are able to use a variety of non-count-by-one strategies to solve addition problems.



Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

How?

Prepare a spinner displaying numerals one to ten and a "flip counter". To make the flip counter, on one side of a counter write "+1" and on the other side write "-1'.

Provide the students with a strip of paper on which to record five numbers in the range 1– 21. Students take turns to spin a number on the spinner. They then double the number to find the answer. If the student has this number on their paper strip they may cross it off. If the doubled number is one more or one less than a number on their paper strip, the student may choose to toss the "flip counter". The winner is the first person to cross off all five numbers.

Variations

All students in the group may cross off the answer if they have it on their paper strip.

Ask the students to write the five numerals vertically down a piece of paper. When the answer has been calculated the student records the number sentence next to the answer.

Students record three numbers on a paper strip instead of five.

Play as two teams before having the students play independently.

If a spinner is not available, use cards 1–10 or a ten-sided die.

Why?

Using knowledge of doubles and near doubles is an efficient strategy for solving some addition and subtraction problems.

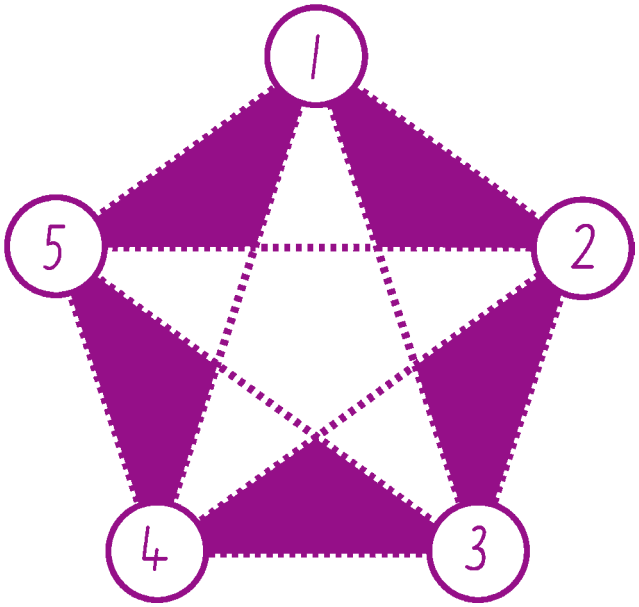
Addition star

Where are they now?

Students are able to solve addition and subtraction questions by counting by ones.

Where to next?

Students are able to use a variety of non-count-by-one strategies to solve addition problems.



TP

After playing the game, ask the students if they had any strategies for winning. If the students discover a strategy for always being able to bridge to the next decade, have them devise a rule to block the strategy.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

BLM

Addition star, page 137

How?

Prepare a copy of *Addition star* BLM for each pair of students. The students will also need a counter and two dice. The students roll the dice and use the numbers that are rolled to indicate the target number. For example if a 5 and a 3 are rolled the students may choose to make the target number 53 or 35. Once the target number has been decided, the first player rolls one of the dice again and places the counter on the corresponding numeral on the addition star. If a "six" is rolled, the player may place the counter on any of the numerals. The second player then moves the counter along any line to add another number to the tally. If a player is able to add a number that bridges the total to the next decade, they have another turn. For example, student A starts at "five". Student B moves to "one" and states the total, *Six!*. Player A moves the counter to "four" and states the total, *Ten! I made it to the decade so I have another turn!* The game continues until one player reaches the target number.

Variation

Start at the target number and subtract from the tally on each move. If a player moves down to the next decade, they have another turn.

Why?

Students need to know a variety of counting strategies to use and apply the most efficient strategy when solving arithmetical problems.

Students record the target number and the additions.

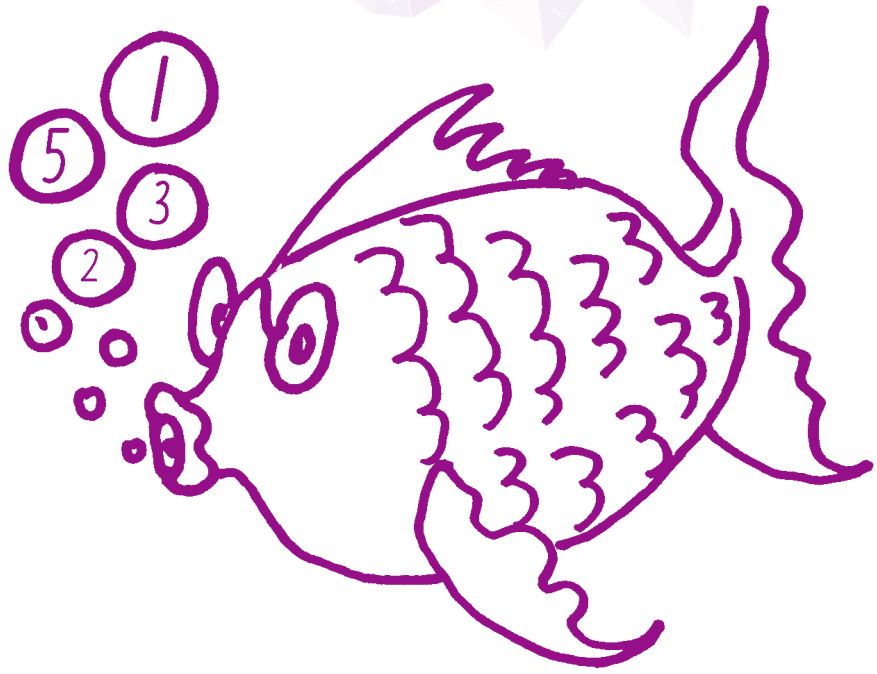
Brainy fish

Where are they now?

Students are able to use counting-on as a strategy to solve addition problems.

Where to next?

Students are able to use a variety of strategies to solve addition problems including doubles, near doubles and combining numbers.



Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

BLM

Brainy fish, page 138

Brainy fish spinner, page 139

How?

Prepare a baseboard using *Brainy fish* BLM and a spinner displaying the following instructions: “Double it”, “Double it plus one”, “Double it take away one”, “How many more to make 10?” (Brainy fish spinner).

Organise the students into groups or pairs and provide them with a fish baseboard, a die and a supply of counters. Each student will need his or her own colour counters. Have the students take turns to firstly roll the die, then spin the instruction spinner. After following the instructions on the spinner, the student determines the answer and places his or her counter onto a corresponding numeral on the baseboard. More than one counter may be placed on a numeral. The activity continues until one student is able to place three counters in a row.

Why?

Students need to be able to apply a range of non-count-by one strategies in order to solve problems efficiently.

TP

Use transparent counters so the numerals may be seen.

Addition wheel pairs

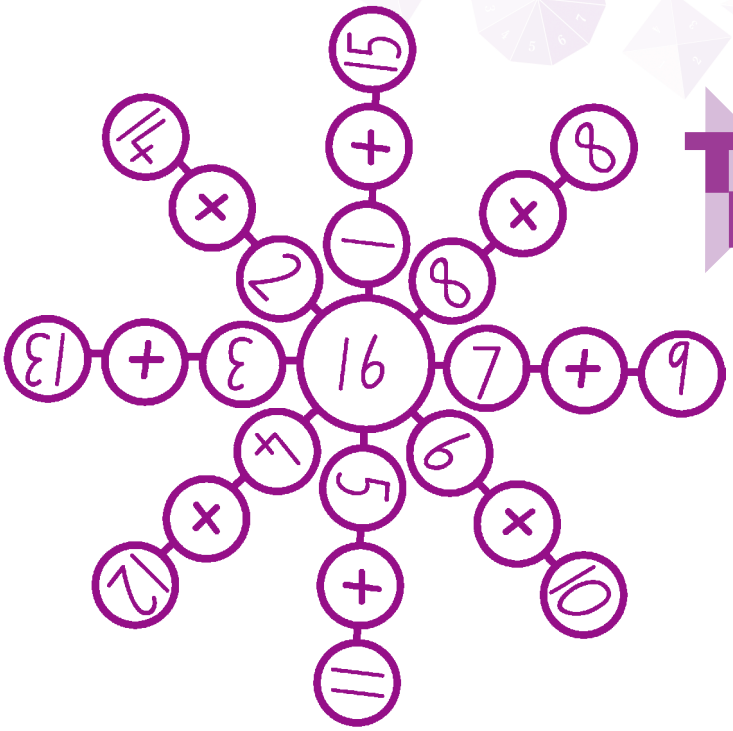
Where are they now?

Students recall some double facts to 20.

Students count by ones when solving addition and subtraction problems.

Where to next?

Students are able to relate doubles to other number combinations.



TP

Depending on the chosen double fact, not all spokes on the addition wheel may be needed or more spokes may be needed for a number larger than the example.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping

Combining and partitioning: levels 1, 2

Recording symbols

BLM

Addition wheel pairs, page 140

How?

Provide the students with a copy of the addition wheel worksheet. Ask the students to nominate a "double" fact they know where the answer is bigger than ten. The students then write the total for the double fact on the centre of the wheel and the "doubles" combination on one of the spokes. Have the students add "one" to one of the numbers and take away "one" from the other number so that the total remains the same. The students then record the new number sentence on the next spoke of the wheel. Continue adding and subtracting "one" from the number sentence until all the spokes are filled. On the second wheel ask the students to add "ten" to the centre number and determine the addition combinations using the first wheel to help them. Discuss the similarities between the two wheels.

Variation

Ask the students to find partners who used the same number of spokes on the addition wheel and compare addition pairs.

Why?

Students need to develop a range of non-count-by-one strategies such as doubling and using known facts to derive an answer.

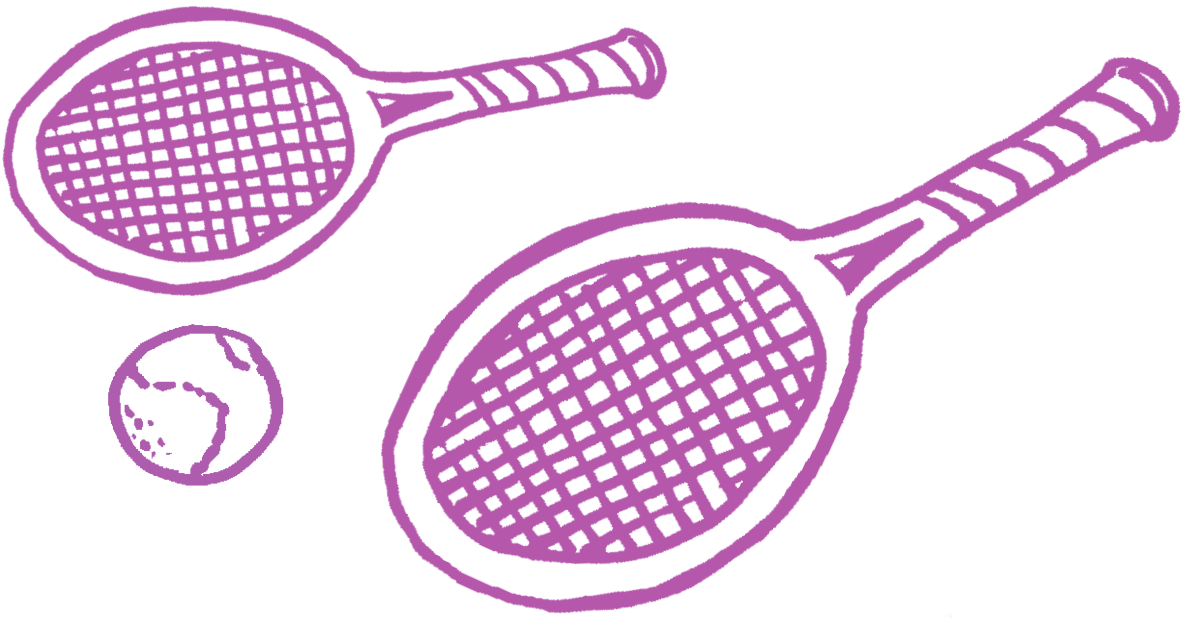
Singles or doubles?

Where are they now?

Students are able to use counting-on as a strategy to solve addition problems.

Where to next?

Students are able to use a variety of strategies to solve addition problems including doubles, near doubles and combining numbers.



Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

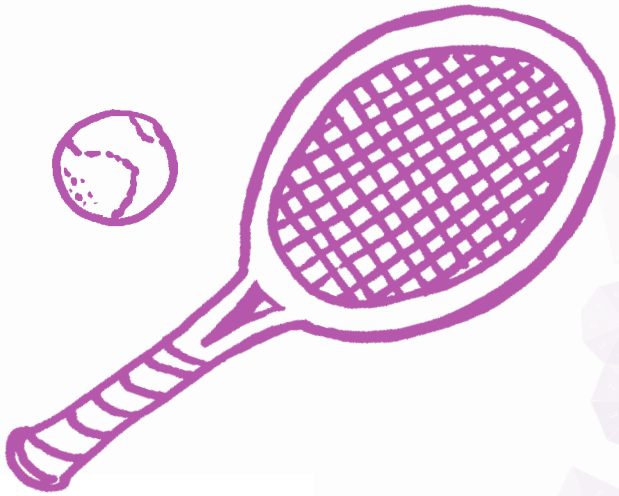
Building addition and subtraction through grouping: facile counting strategies

How?

Prepare two dice. one displaying numerals 1– 6 and the other marked S, S, S, D, D, D. “S” means the number rolled on the other dice remains as a “single” number. “D” means the number rolled on the other dice is doubled. Each student takes a turn to roll the dice and keeps a tally of his or her score. The first player to reach 100 is the winner.

Variation

Start with a score of 100 and subtract the rolled number.



Why?

Students need to develop a range of non-count-by-one strategies such as doubling and using known facts to derive an answer.

Even Stevens

Where are they now?

Students are able to use counting-on as a strategy to solve addition problems.

Where to next?

Students are able to use a variety of strategies to solve addition problems including doubles, near doubles and combining numbers.

Encourage the students to explain their solutions. Building numbers to 10 and 20 and doubling are useful strategies to model.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

Combining and partitioning

Recording symbols

How?

Prepare nine cardboard squares and write the number "one" on three cards, the number "four" on three cards and the number "sixteen" on the remaining three cards. Place the cards into a box with a lid. Instruct the students to write the even numbers to 62 on a piece of paper. Have one of the students take a turn to shake the box and then turn it up so the cards fall to the floor. The student then adds up any cards that have landed face-up and if the sum is on his or her paper, crosses it off. The first player to cross off ten different numerals wins.

Variations

Have the students determine all of the numbers that can be created using the cards, prior to playing the game.

Students construct bingo boards with some of the even numbers to fifty recorded on each student's board.

The first player to cross off five different numerals wins.

Provide the students with a hundred-chart. After the student has added the cards, he or she crosses off the corresponding number on the hundred-chart.

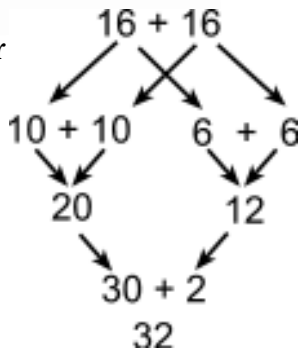
Use popsticks instead of numeral cards.

Why?

Students need to develop a range of non-count-by-one strategies such as *combining and partitioning* to derive an answer.

WR

When discussing strategies for solving addition problems, opportunities will arise to record students thinking. For example, a student may use a collection-based strategy for adding 16 and 16.



Engineer's dice

Where are they now?

Students are able to complete calculations using the four operations.

Where to next?

Students use a variety of mental strategies to solve problems involving the four operations and use their understanding of number concepts in a flexible way.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

NS2.3: Uses mental and informal written strategies for multiplication and division

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

Combining and partitioning

Building multiplication and division through equal grouping: level 5

Recording symbols

How?

Provide each group of students with five dice. To play the game a target number is selected by the group. The students then take turns to roll the dice in the following way:

- Roll all five dice. Choose two of the dice and nominate an operation (+ - x ÷) to carry out with the numbers rolled. Record the result. Discard these two dice.
- Roll the remaining three dice. Choose one number rolled, complete another operation (+ - x ÷) with the chosen number and the first score. Discard that die.
- Roll the remaining two dice. Choose one number rolled and complete the same process as the step above using the current total.
- Roll the last die and complete the same process using the current total.

After each player has had his or her turn, the students compare their totals to see who is closest to the target score.

Variation

Change the operations that can be used. For example, doubling plus one.

Why?

Being able to calculate mentally is often quicker and easier than formal algorithms and can lead to a better understanding of number concepts such as place value and numerical operations.

Fancy dice

Where are they now?

Students are able to solve addition and subtraction questions by counting by ones.

Where to next?

Students are able to use a variety of non-count-by-one strategies to solve addition problems.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

Combining and partitioning

Recording symbols

How?

Provide each group with five dice. Each student takes it in turn to roll the dice and add the total. The student continues to roll the five dice and accumulate the total unless a "two" or a "five" is rolled. If so, any dice displaying a "two" or a "five" must be taken out for all subsequent throws for that player. The student throws the remaining dice again and keeps going until he or she has no dice left. If "six" is rolled on two of the dice, the player loses all of the score for that turn and it is the next player's turn. If "six" is rolled on three dice, the player loses all of his or her score, returning to zero and it is the next player's turn. The first player to reach 200 wins.

Variation

Each player begins with a score of 200 and the total is subtracted from 100. The first player to reach zero is the winner.

Why?

Being able to calculate mentally is often more practical than completing formal algorithms. Students need to develop a range of mental strategies to apply to problem-solving situations.

WR

Each student will need to keep his or her own accumulating total. Have each student demonstrate to the group how the addition or subtraction was calculated.

Counter play

Where are they now?

Students are able to solve addition and subtraction questions by counting by ones.

Where to next?

Students are able to use a variety of non-count-by-one strategies to solve addition problems.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building addition and subtraction through grouping: facile counting strategies

Combining and partitioning

Recording symbols

BLM

Counter play, page 141

How?

Organise the students into pairs and provide each pair with a copy of *Counter play* BLM, seven counters of one colour, say red, and one counter of another colour, say blue, and paper and pencil for scoring. Have the students lay out the counters so that the "blue" counter is on the top left hand corner of the grid and the "red" counters are on all other squares except the bottom right-hand corner. This corner does not begin with a counter on it. The aim is for the students to move the "blue" counter to the opposite corner keeping to the following rules:

- All moves must be vertical or horizontal.
- Only one counter must be on a square at any time.
- Take it in turns to move a counter.
- A player can only move one space at each turn.
- A player cannot uncover the same number twice in a row.

Players keep score by adding the number on the square the counter was moved from to their total. The player with the lowest score, when the "blue" counter is placed on the 6, wins.

Why?

Being able to calculate mentally is often more practical than completing formal algorithms. Students need to develop a range of mental strategies to apply to problem-solving situations.

Have the students record and explain their methods for adding.

Copy that

Where are they now?

Students need to count each item to find the total.

Where to next?

Students are able to instantly recognise and state the total number of items in a small group.

Syllabus outcomes

NS1.1: Count, orders, reads and represents two- and three-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: perceptual

BLM

Copy that, page 142

How?

Prepare dot pattern flash cards for numbers up to seven (*Copy that* BLM). Provide each student with a pile of counters. Flash a domino pattern card to the students for about a second. Ask them to use their counters to reproduce the pattern. Discuss how many dots are in the pattern and how they remembered what the pattern looked like.

Variations

Ask the students to hold up the same number of fingers as the total of the dot pattern.

Use alternative material such as modelling dough to reproduce the dot patterns.

Display dot patterns on an overhead projector using transparent counters or cardboard with patterns cut-out or holes punched.

Why?

Recognising patterns instantly will assist students, to develop visualisation of numbers so as not to have to rely on counting perceived items. This contributes to early forms of grouping.

Using random patterns

Where are they now?

Students need to count each item to find the total.

Where to next?

Students are able to instantly recognise and state the total number of items in a small group (up to 6 or 7).

TP Make an art display with the dot patterns at the end of the activity.

Syllabus outcomes

NS1.1: Counts, orders, reads and represents two- and three-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

Subitising: perceptual

How?

Have the students draw dot patterns onto cardboard squares for numbers three to eight. Tell them they are to draw a dot pattern for a different number on each card and that the dots can be in any arrangement they choose. With the overhead projector turned off, place a number of counters (from three to eight) onto the screen. These may be in random or traditional dot patterns. Briefly show the counters on the overhead. Each student then holds up a card showing the same number of dots as counters displayed on the overhead. Discuss how many counters were shown. Compare dot patterns.

Variations

Have students call out the number of counters on the overhead.

Have students call out how many to make ten.

Use the cards to play "memory match" games. (Matching a standard dot pattern to a random pattern.)

Why?

Recognising patterns instantly will assist students to develop visualisation of numbers so as not to have to rely on counting perceived items. This contributes to early forms of grouping.

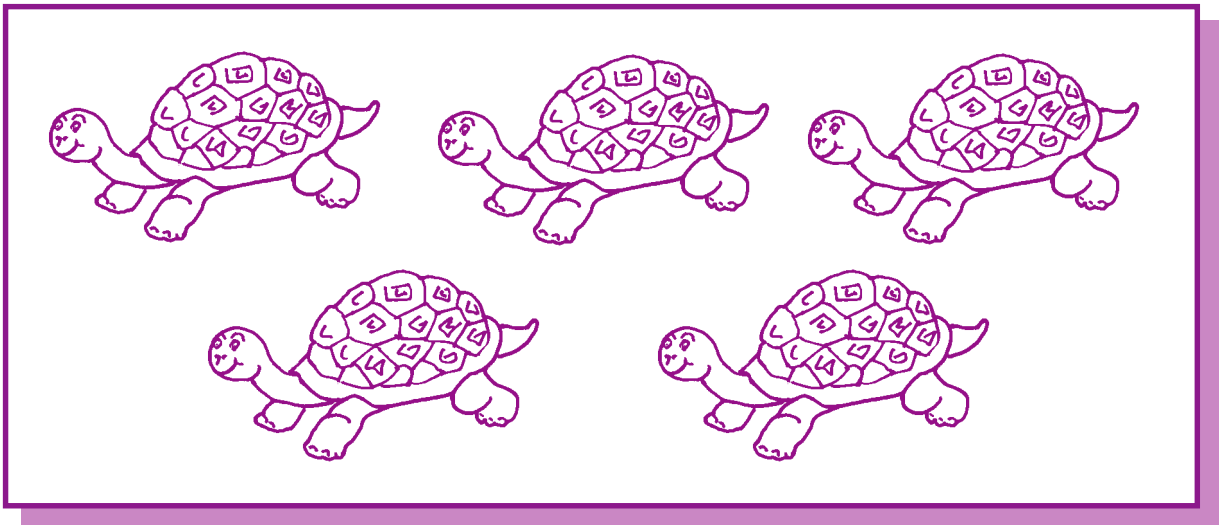
Matching ten-frames

Where are they now?

Students need to count each item to find the total.

Where to next?

Students are able to instantly recognise and state the total number of items in a small group.



Syllabus outcomes

NS1.1: Counts, orders, reads and represents two- and three-digit numbers

WMS1.2: Uses objects, diagrams, imagery and technology to explore mathematical problems

CMIT reference

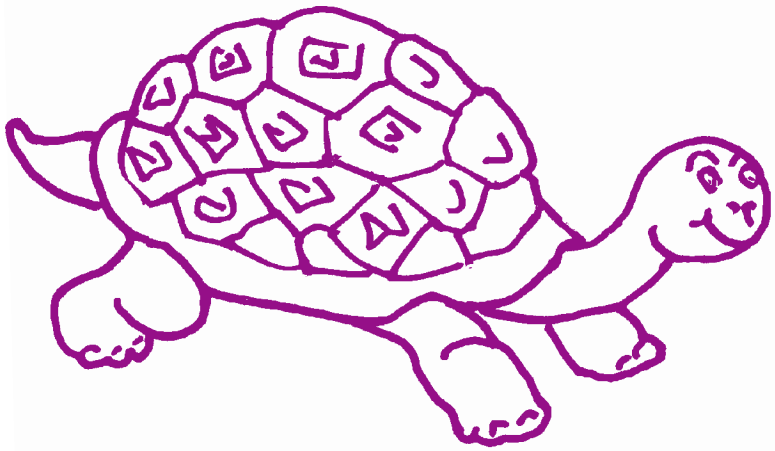
Subitising: perceptual

How?

Construct a set of flash cards displaying both random and standard dot patterns for numbers 1–7. (Cards could also be made of standard dot patterns for numbers 8 –10.) Provide the students with a set of ten-frame cards displaying dots 1–10. Flash one of the dot pattern cards briefly. Have the students select a ten-frame card to match the number of dots on the flash card.

Variation

Replace the dots on the flash cards with simple pictures.



Why?

Recognising patterns instantly will assist students, to develop visualisation of numbers so as not to have to rely on counting perceived items. This contributes to early forms of grouping.

Bunches of five

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

CMIT reference

Combining and partitioning: To 10

Five as a sub-base

BLM

Bunches of five, page 143

Double scoops, page 144

How?

Provide each student with a copy of *Bunches of five* BLM. Each group will also need a die or spinner. Have the students take turns to roll the die. The student selects one of the pairs of hands on the worksheet and records the number underneath the hand(s). On the students next turn to roll, he or she may again choose any set of hands to add to. However, the student must roll the exact number to make ten to complete a set of hands. The winner is the first person to complete all sets of hands on the worksheet.

Variation

If needed, have students colour in the corresponding number of fingers for each turn.

Using the *Double scoops* BLM, the student rolls the die and records the number on a cone so that the two numbers added together equal ten.

Have six students out the front of the room to model the activity. As you roll and select a pair of hands, have the student put down the required number of fingers.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

"Make ten" grids

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

CMIT reference

Combining and partitioning: To 10, To 20

BLM

"Make ten" grids, page 145

How?

Organise the students into pairs. Provide each pair of students with a "Make ten" grid and a ten-sided die showing numerals 0–9. Have the students take turns to roll the die and call the number rolled. The student then states the number needed to make ten, finds the number on the grid and marks it with a cross or circle. Play continues until one player is able to mark off four numbers in a row.

Variations

Have the students create their own grids by writing numerals onto the blank grid.

Change the numbers on the grid so that the student makes combinations to equal twenty.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

Memory of tens

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

CMIT reference

Combining and partitioning

How?

Prepare two sets of cards in the range 0–10. Ensure each set is on different coloured card. Place the cards face down in front of the students. Have the students take turns to turn over a card from each set, trying to turn over two cards that combine to make 10. If the student is successful, they keep the pair of cards.

Replace one set of cards with a set of cards in the range 20–30. Have the students take turns to try to find two cards that combine to make 30.

Variations

Change the range of cards to 0–20.

Use ten-frame cards to represent numbers 0–10.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

Have the students record the number combinations that equal ten. Discuss how the students know that they have recorded all possible combinations. Then have the students record the combination that make 20 and 30.

Couple cups

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

TP

Note the strategies students use to tally the numbers.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

PAS1.1: Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships

CMIT reference

Combining and partitioning: To 10

How?

Decorate ten paper cups with bright patterns. Place the cups on the floor and without the students seeing, hide a different number, from 1–9 (not in sequence) of plastic teddies, under each cup. Two cups will need five teddies under each. Teddies could be substituted with marbles, counters, buttons or other suitable material. When the material is ready have the students take turns to lift two cups. As soon as the number of teddies under the first cup has been determined, the student is to state that number and the number needed to make ten before lifting the second cup. If the total is ten the student keeps the teddies and returns the cups. If the total is not ten, the student replaces the cups over the teddies.

Variations

Record the number combinations.

If the student chooses two cups that add to ten, they replace the cups and the teddies in a different location. When they put the teddies back they must share the ten teddies in a different combination from the one they found. The same cups may not be chosen for at least two turns.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

Nine piles

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

PAS1.1: Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships

CMIT reference

Combining and partitioning: To 10

How?

Remove the "picture" and "ten" cards from a deck of playing cards. For this activity ensure the students know that the "Ace" is equivalent to "one". Deal out the cards face up into nine piles. The students take turns to locate two cards that total to ten. If able to find two cards equalling ten, the student removes and keeps the cards, revealing two new cards. The activity continues until a player is unable to pair-up two cards that total ten.

Variations

For this activity tell the students that the "Ace" is now equivalent to "eleven" for this activity and have them locate and remove cards that total twenty. This may be two or more cards at one turn.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

Number chop

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

PAS1.1: Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and build number relationships

CMIT reference

Combining and partitioning: To 20

Recording symbols

How?

Provide the students with 20 "unifix" cubes. Ask the students to choose a number between 11 and 20 and to link a corresponding number of cubes together. Tell the students to "break" the cubes into two groups and record the combination. Have the students determine and record all possible combinations. Next have the students complete the same process for a different number without using the unifix cubes.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

WR

Have the students record the addition combinations and then determine how the numbers could be used to show subtraction facts.

Number draughts

Where are they now?

Students solve addition and subtraction tasks using count-by-ones strategies.

Where to next?

Students use grouping strategies to solve addition and subtraction tasks.

Syllabus outcomes

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

PAS1.1: Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships

CMIT reference

Combining and partitioning: To 20

BLM

Number draughts, page 146

How?

Prepare a 5 x 4 grid baseboard and a set of numeral cards showing numbers 1–9 and 11–20 and a star card for each pair of students. Have the students shuffle the cards and place them in random order face-up onto the 20 sections of the baseboard. To play, the students take turns to move one card vertically, horizontally or diagonally one space, onto another card with the aim of making a total of 20 with the two cards. If the cards total 20 the player keeps the cards. If a pair cannot be made the player may move any card into an adjoining space in preparation for a later move.

The "20" and "✪" cards are "wild". The "20" card may be picked up on its own and kept but can only be taken if no other pairs can be made. The "✪" card may represent any number. The student may pair it with any adjoining card. However, it can only be used if no other pairs can be made and the student using it must indicate the value it takes.

Variation

Students move a card by jumping over another card vertically, horizontally or diagonally.

Why?

Developing an understanding that numbers can be regrouped into parts is essential if students are going to move beyond counting-by-ones as a counting strategy.

Count-off

Where are they now?

Students know the sequence of counting forwards and backwards by tens.

Where to next?

Students are able to count forwards and backwards by tens, both on and off the decade.

Syllabus outcomes

NS1.1: Counts, orders and represents two- and three-digit numbers

CMIT reference

Counting by 10s and 100s: level 2

Recording symbols

How?

Roll a ten-sided (decahedron) or a twelve-sided (dodecahedron) die. Have the students start counting from the number rolled, adding ten to the count each time up to the 90s. Then count backwards by tens.

Display a hundred chart to the students. Have one student select a number from 1–9 on the hundred chart and call out the number. Once the student calls out the selected number, the rest of the class continue counting by adding ten each time. The first student may continue to locate each number after it has been called.

Why?

Developing knowledge of forwards and backwards counting skills will assist students in using mental calculations to solve two-digit addition and subtraction tasks.

WR

The "empty number line" could be used to record student's thinking and to demonstrate building-on by tens.

Number line counting

Where are they now?

Students know the sequence of counting forwards and backwards by tens.

Where to next?

Students are able to count forwards and backwards by tens, both on and off the decade.

Syllabus outcomes

NS1.1: Counts, orders and represents two- and three-digit numbers

CMIT reference

Counting by 10s and 100s: level 2

How?

Display a 0–100 number line to the students. Ask a student to nominate a single-digit number from which to begin counting. Encourage the students to count along the line for ten counts from the nominated number. Attach a peg, or paperclip, to the last number of the count. Continue by counting on ten more each time and marking the last number counted. Chant the sequence of "marked" numbers. Repeat the process, starting from a different single-digit. After a few turns, discuss other sequences without having to mark each number first.

Why?

Developing knowledge of forwards and backwards counting skills will assist students in using mental calculations to solve two-digit addition and subtraction tasks. Students need to be able to count by tens from the middle of the decade to use the "jump" method to solve addition problems. This involves starting from one number and adding on by tens and ones.

Show how these findings could be represented on an "empty number line".

Hands up

Where are they now?

Students know the sequence of counting forwards and backwards by tens.

Students use their fingers to count on by ones when solving addition problems.

Where to next?

Students are able to count forwards and backwards by tens, both on and off the decade and use this strategy to solve addition problems.

TP

This activity could be used to demonstrate the *jump method* of adding two, two-digit numbers.

Jump method: When adding two numbers, the student starts counting from one number and adds firstly the "tens" and then the "ones" from the second number to find the total.

Syllabus outcomes

NS1.1: Counts, orders and represents two- and three-digit numbers

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one-, and two-digit numbers

CMIT reference

Counting by 10s and 100s: level 1 and 2

Building place value through grouping: level 1

Finger strategies

Recording symbols

How?

Ask a student to come to the front of the class and hold up ten fingers. Then ask the student to demonstrate a number such as "43" using fingers. If the student is hesitant, suggest that friends may help in the demonstration by raising their fingers as well. Ask the class to check the number of fingers by counting groups of tens and then adding the ones. Then ask the class to check the number again, this time by counting from the "ones" first and then counting on by "tens". In the example of "make 43" the counting sequence would be 10, 20, 30, 40, 41, 42, 43 and then 3, 13, 23, 33, 43. Repeat with various other numbers. When the class is confident in representing numbers in this way, expand the activity to representing two numbers and adding them together.

Variation

Have one student represent a two-digit number using as many students' hands as needed, without stating what the number is. Each member of the class then determines and records the number.

Why?

Developing knowledge of forwards and backwards counting skills will assist students in using mental calculations to solve two-digit addition and subtraction tasks. Students need to be able to count by tens off the decade to be able to use the "jump" method for solving addition problems.

WR

When students are using fingers to form a number, record the numeral on the chalkboard and discuss the number of tens and ones and how they are used to form the number.

Have students use informal recordings on the chalkboard to demonstrate their methods of solving the additions.

An empty number line could be used to record the *jump* procedure.

Sticks of ten

Where are they now?

Students know the sequence of counting forwards and backwards by tens.

Students use their fingers to count on by ones when solving addition problems.

Where to next?

Students are able to count forwards and backwards by tens, both on and off the decade and use this strategy to solve addition problems.

Syllabus outcomes

NS1.1: Counts, orders and represents two- and three-digit numbers

NS1.2: Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

CMIT reference

Counting by 10s and 100s: level 1 and 2

Building place value through grouping: level 1

Recording symbols

How?

Ask two students to come to the front of the class. Have ten sticks of ten unifix cubes and give five sticks to one student and five to the other. Ask one of the students to break off some of the cubes from one of the sticks and give it to the other student. Both students display their sticks. Have the class count the number of cubes the first student has and then use the second student's sticks to count on by tens and then ones to reach 100.

Variations

Once the class has determined how many cubes the first student has, ask them to work out how many the second student has without seeing the cubes.

Have the first student break off more than ten cubes and give them to the second student.

Why?

Developing knowledge of forwards and backwards counting skills will assist students in using mental calculations to solve two-digit addition and subtraction tasks. Students need to be able to count by tens off the decade to be able to use the "jump" method for solving addition problems.

Have the students record their working when solving the missing addend task.

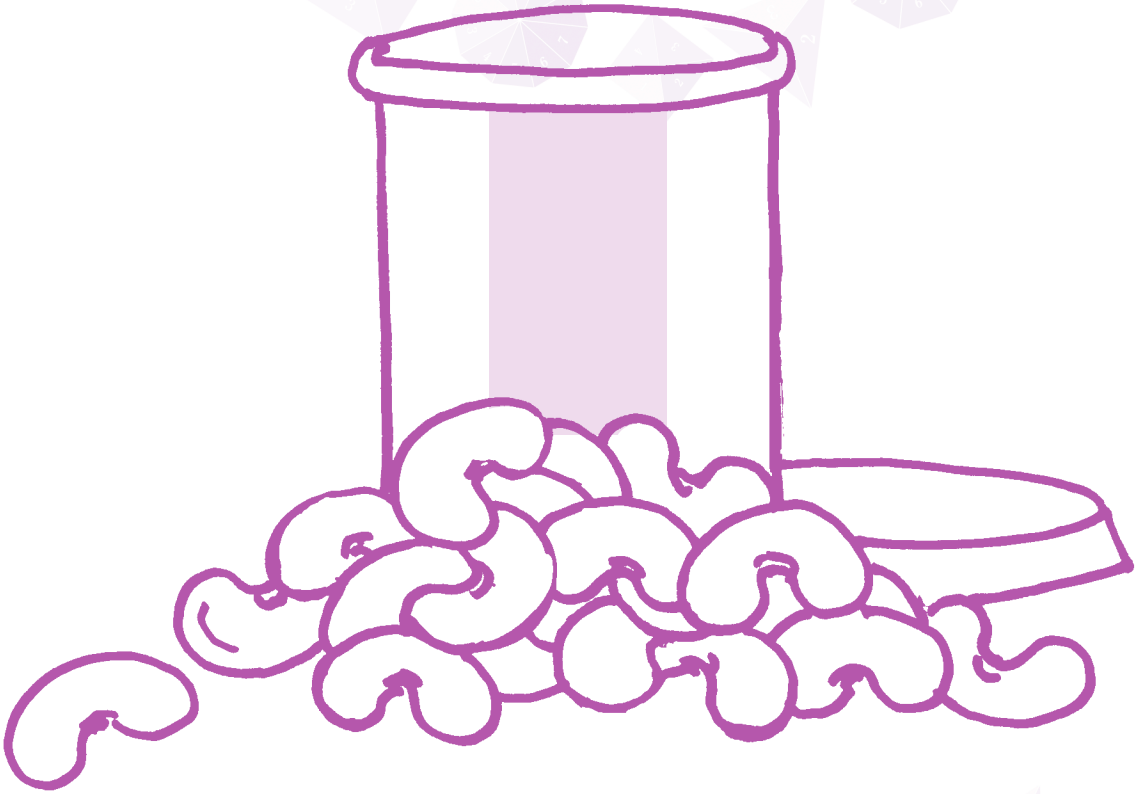
Bean counter

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.



Syllabus outcomes

NS2.1: Counts, orders, reads and records numbers up to four digits

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

How?

Present the students with a large quantity of dry beans or other similar material that can be stored in small containers. Ask students to estimate the number of beans. Have the students count out ten beans and place the beans into a container such as a film canister. Students continue until all beans have been placed into groups of ten. Encourage the students to find the total number by counting by tens and then adding any remaining single units.

Variation

Count the remaining beans first and then count on by tens to find the total.

Why?

Reorganising single units into groups of "ten" assists students to see ten as a composite unit. This understanding will aid students' knowledge of place value.

Record the total number for each material. Discuss the unit structure of ones, tens and hundreds. For example, if the total is 102, record how many tens and units make up the total.

Dizzy dots

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

TP Lead the students to counting by fives and tens.

Syllabus outcomes

NS2.1: Counts, orders, reads and records numbers up to four digits

PAS2.1: Generates, describes and records number patterns using a variety of strategies and completes simple number sentences by calculating missing values

WMS2.4: Checks the accuracy of a statement and explains reasoning used

CMIT reference

Building place value through grouping: level 1

BLM

Dizzy dots, page 147

How?

Present the students with various large arrays arranged in rows of five or ten. A 10 x 10 array is included in the BLM section. This can be used to create different array patterns, e.g. 5 x 10. Ask the students to think of a way to count the dots quickly and easily to determine the total. The hundred-chart could be used to assist students with counting.

Why?

Reorganising single units into groups of "ten" assists students to see ten as a composite unit. This understanding will aid students' knowledge of place value.

Building numbers with ten-frames

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

Make ten ten-frame cards from the blank BLM. Leave one card blank and the rest to represent a number from 1–9.

Syllabus outcomes

NS2.1: Counts, orders, reads and records numbers up to four digits

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

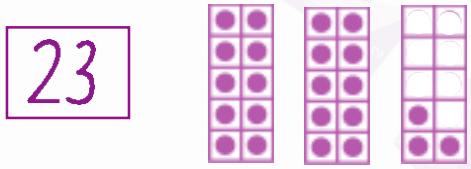
Numeral identification: level 3

BLM

Ten-frame, page 148

How?

Present the students with two piles of numeral cards displaying numerals 0–9 and a supply of ten-frame cards. The students will require nine "full" ten-frame cards and one of each ten-frames showing 1–9 dots. Have the students draw a numeral card from each pile and construct a two-digit number. The students then represent the numeral using the ten-frame cards.



Ask the students to indicate how many more are needed to reach the next decade.

Variation

Provide the students with additional ten-frame cards and numeral cards. Pose addition problems for the students to solve. Have the students use the ten-frame cards to represent the numbers and to solve the problems.

Why?

Part of the sequence of developing an understanding of place value moves from students seeing ten as a composite unit to students using tens and ones to find the total of two, two-digit numbers.

WR

The ten-frames could be used to introduce written algorithms by writing the number the ten-frame represents under each frame.

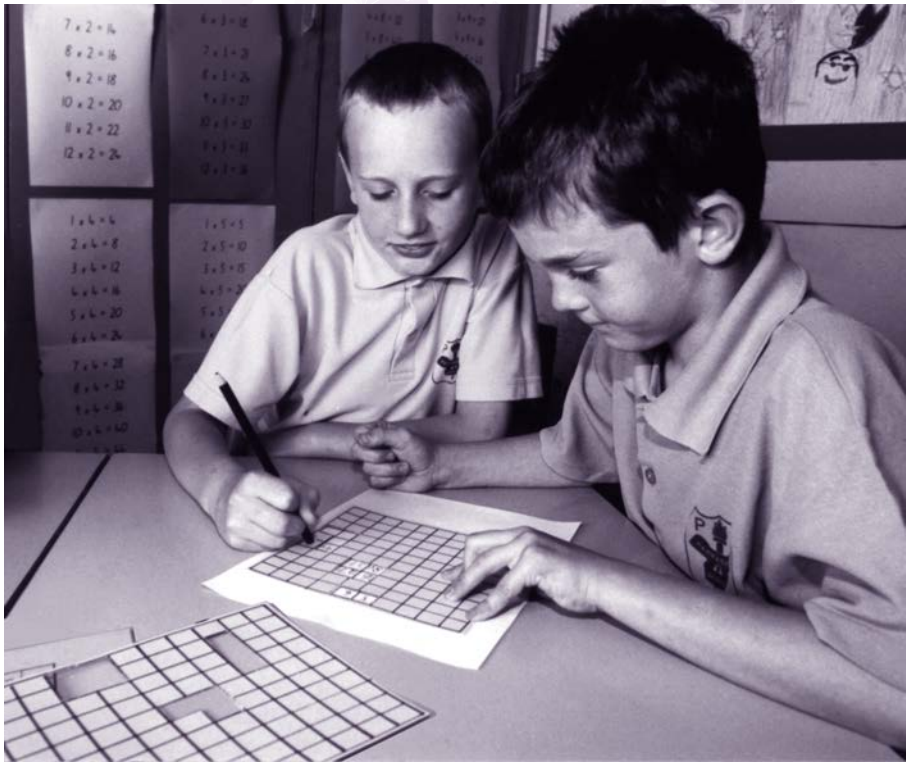
Hundred chart windows

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.



Syllabus outcomes

NS2.1: Counts, orders, reads and records numbers up to four digits

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

Numeral identification: level 3

BLM

Hundred chart windows, page 149

Hundred chart windows sample, page 150

How?

Provide the students with a hundred chart grid. Prepare a second hundred chart grid card which will fit onto the first card. Cut "windows" out of the second grid card. For example, cut out a window covering four squares on the grid. Have the students place the "window card" on top of the grid and using the grid to assist them, determine which numbers on the hundred chart should be recorded in each square of the window. Three or four windows should be cut out of each "window card". Have the students rotate the window card 90º to reveal new boxes on the hundred chart grid. Students need to rotate the card three times and repeat the process of recording the missing numbers. An adequate supply of window cards will need to be prepared, enabling all numbers to be recorded on the grid. (An example of window cards is included in the BLM section.)

Variation

Have a card with only one square cut out and place it on top of a hundred chart. Instruct the students to place the card on a starting number and then to add on a nominated amount by moving the card down by tens and across by ones.

Why?

Reorganising single units into groups of "ten" assists students to see ten as a composite unit. This understanding will aid students' knowledge of place value.

TP

Practice locating numbers on a 1–100 chart by counting by tens and ones prior to this activity.

Money problems

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

TP

After practising counting by tens, on and off the decade, move to hundreds and tens, on and off the decade.

Syllabus outcomes

NS2.1: Counts, orders, reads and records numbers up to four digits

NS2.2: Uses mental and written strategies for addition and subtraction involving two- and three-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Counting by 10s and hundreds: level 2

Building place value through grouping: level 1

How?

Provide the students with a collection of ten-cent coins (or play money). Nominate a starting amount, say 30 cents, and ask the students to use the coins to determine how much more is needed to equal $1.00.

Variations

Nominate a starting amount that is off the decade, say 25 cents. Provide the students with the necessary coins to enable them to count by tens and then bridge to one hundred to determine the amount needed to total $1.00.

Present the student with six, five-cent coins, five, ten-cent coins, four, twenty-cent coins and two, fifty-cent coins. Ask the students to determine as many different ways as they can to make up $1.00 using the coins. Have the students record the combinations.

Why?

Knowledge of forward and backward counting sequences by tens, both on and off the decade, will assist students with mental strategies when solving addition and subtraction problems.

Four turns to 100

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

TP Demonstrate the use of the empty number line prior to this activity.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

How?

Organise the students into groups of four. Provide each group of students with a pack of cards in the range 1 to 9. Each player draws a card from the deck and decides if the number they have drawn will represent ones or tens. For example, if a five is drawn it can represent five or fifty. The players take a second draw from the pack, again nominating if the number represents tens or ones and adds the number to their first card. Have the students record their total on an empty number line. Continue the activity until each student has drawn four cards. The player with the highest total not exceeding 100 wins.

Variations

Players start at 100 and subtract the numbers, after nominating if the number drawn represents tens or ones. The player closest to zero is the winner.

Players draw two numbers from the pile and make the highest two-digit number possible. This becomes their starting number and they continue to play as in the above variation.

Why?

In solving addition and subtraction problems, students need access to a range of strategies other than counting by ones.

Eggsactly

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

BLM

Eggsactly, page 151

How?

Prepare empty egg cartons by writing a numeral in the range 0–9 inside each of the cups. Place two counters into the egg carton. Provide each group or pair of students with the prepared egg carton, a pack of cards displaying numerals 0–9 and a supply of tens-strips and single-unit strips. Have the students take turns to shake the carton and open it to see which two numbers the counters have landed on. The student decides in which order the numbers will be used to make a two-digit number and then represents the number using the number cards. The student repeats the action and this time represents the two-digit number using the ten-strips and single-dot strips. The student then adds the two numbers together to determine the total and records the answer. Encourage the students to state how they added the numbers. The partner or group members should agree with the total before it is recorded.

Why?

In solving addition and subtraction problems, students need access to a range of strategies other than counting by ones.

Cover-up strips

Where are they now?

The student counts on by ones when solving two-digit addition questions. The student does not treat ten as a composite unit for counting but rather as ten single units.

Where to next?

The student treats ten as a composite unit and can solve two-digit addition and subtraction questions by counting by tens and ones.

TP

Encourage the students to tell their partner how they are combining the numbers.

Syllabus outcomes

NS2.2: Uses mental and written strategies for addition and subtraction involving two-, three- and four-digit numbers

WMS2.2: Selects and uses appropriate mental or written strategies, or technology, to solve problems

CMIT reference

Building place value through grouping: level 1

Combining and partitioning

BLM

Hundred chart, page 152

Cover-up strips, page 153

How?

Provide each pair of students with two dice, a hundred chart and a set of "cover-up" strips. To begin the activity the first student rolls the two dice, states the total and represents this total on the hundred chart using the strips. The activity continues with each student taking turns to roll the two dice and adding the number rolled to the previous total. Each time a new total is made the student must represent the total with strips on the hundred chart. This will involve the students regrouping the numbers and replacing some of the strips with appropriate combinations. For example, if a nine was originally rolled and then twelve was added to the total, the student would replace the strips representing the nine with a "twenty" strip and a "one" to represent the new total of twenty-one. Play continues until the entire hundred chart is covered. The last roll must make exactly 100.

Variation

Begin with the hundred chart covered with the strips. Have the students roll the two dice and subtract the amount from 100. The student would then remove the appropriate number of strips. Strips may need to be replaced as new combinations are made.

Why?

In solving addition and subtraction problems, students need access to a range of strategies other than counting by ones.