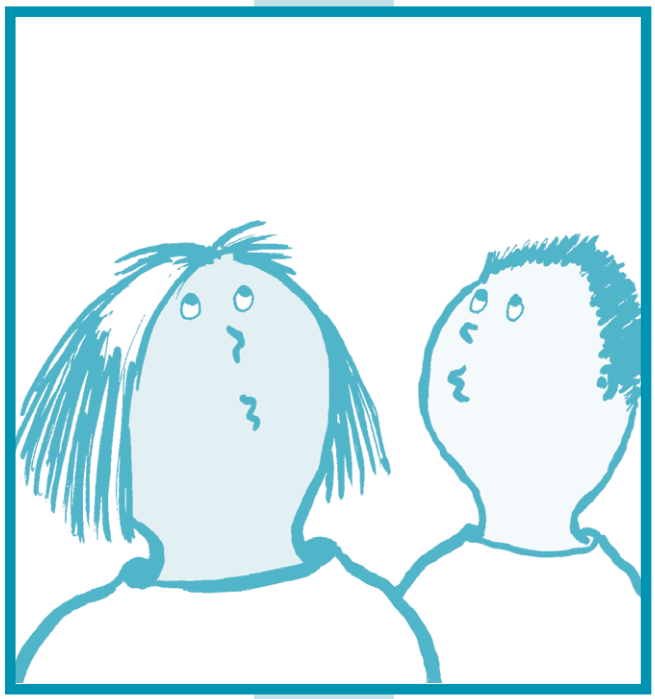
Students at the figurative counting stage



Students at the figurative counting stage

The nature of the learner

Students using figurative strategies to aid counting are able to count collections which are totally or partially concealed. They do not need to see, feel or hear the items in a collection to be able to count the collection. However, typically they rely on the simple strategy of counting by ones, starting from one to find a total. For example, when asked to count a collection of six items and a collection of three items which are both concealed from view, the student would be able to count the nine items often using fingers to represent the items. At the figurative counting stage students have an understanding of numbers as entities. For example, to form six using fingers, a student at this stage will raise six fingers simultaneously. By comparison, at the perceptual counting stage, a student will construct six using fingers by raising them one at a time as they count to six. The total of six and three, however, is still found at the figurative stage by counting from one.

Students at this stage demonstrate an understanding of the conservation of number and can represent a number in a variety of ways through such strategies as separating (partitioning) and combining.

Students are able to consistently produce the correct forward and backward number word sequences in the range from zero to twenty. Many may be able to go beyond this range. They are able to use their knowledge of forward and backward number word sequences to produce the number before and the number after a nominated number.

Students demonstrate their understanding of the numerical value of numbers by naming numerals and labelling collections of up to twenty items.

Students at the figurative counting stage are working towards

- using counting on to solve addition tasks
- using counting down to solve subtraction tasks
- developing base ten knowledge
- forming equal groups and finding their total.

Teaching considerations

When developing teaching and learning programs for students working at the figurative counting stage, teachers need to consider:

- ### Strategy development

  Students working at the figurative stage of counting are unable to draw from a wide range of strategies to solve number problems. Therefore there is a need to model effective strategies. At this stage the main focus for teaching should be on developing the counting on procedure. This will also be a major indicator when assessing students' abilities.

  Ensure that students are given opportunities to practise the strategy of counting on in small-group, pair and individual activities. Be aware that students who demonstrate the ability to use counting on may revert to the strategy of counting from one when they are presented with difficult tasks.

- ### Language development

  As students develop additional problem-solving strategies, ensure that they are taught the explicit mathematical language needed to explain their solutions. Vocalising the procedures used enables students to clarify their thinking and reinforces concepts.

- ### Numeral identification

  Whilst using the arithmetical strategies associated with the figurative stage, students may exhibit different levels of knowledge of numerals. The identification of numerals does not develop uniformly. Students may be able to identify some of the numerals beyond twenty before they can identify all the numerals up to twenty.

Index of activities

Where are they now?

Students do not see ten as a countable unit. The student's focus is on the individual items that form a unit of ten. In addition or subtraction tasks involving tens, students count forwards and backwards by ones.

Where to next?

Students see ten as a unit composed of ten ones and are able to use the unit to count. However, they are dependent on visual representations of ten, such as strips of card showing ten dots or using ten fingers.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1 Counts, orders, reads and represents two- and three-digit numbers

WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained

LFN reference

Base ten: level 1.

How?

Collections

Present students with a large collection of items, such as counters, pebbles, or buttons, a supply of containers, such as patty papers or cups, and a large sheet of cardboard. They will also need two sets of numeral cards ranging from zero to nine. Divide the chart into a "tens" and a "ones" column. Present the collection of items to the students and allow them to count the items. Each time ten items are collected, the students place the items into a container and move the container to the left-hand side of the chart, that is, onto the "tens" column. Students then place a numeral card above the "tens" column, indicating how many groups of ten have been collected. As succeeding tens are collected, students continue to add them to the left-hand side of the chart and replace the numeral card accordingly. Remaining items are placed on the right-hand side of the chart, in the "ones" column. Students then place a corresponding numeral card above the "ones" column to form a two-digit number.

Why?

Developing an understanding of tens and ones will assist students in using strategies other than counting by ones to solve problems.

Where are they now?

Students do not see ten as a countable unit. The student's focus is on the individual items that form a unit of ten. In addition or subtraction tasks involving tens, students count forwards and backwards by ones.

Where to next?

Students see ten as a unit composed of ten ones and are able to use the unit to count. However, they are dependent on visual representations of ten, such as strips of card showing ten dots or using ten fingers.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1 Counts, orders, reads and represents two- and three-digit numbers

WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Base ten: level 1.

How?

Bundling

Present the students with large collections of popsticks. Have the students bundle the popsticks into groups of ten and place any remaining popsticks to the side of the bundles. Encourage students to count by tens to find the total and add on any remaining popsticks. Students should then label the collection using numeral cards. Interlocking blocks, such as Multilink, Unifix or Centicubes, could also be used.

Trading game

Supply students with a collection of base ten material. The students take turns to throw a die and take a corresponding number of base ten "shorts" from a central pile. On succeeding throws of the die, students add appropriate numbers of "shorts" to their collection. As the students collect ten "shorts" they swap or trade them for one base ten "long". Continue the activity until one, or all students, can trade ten "longs" for a base ten "flat".

Counting on

Prepare numeral cards in the range eleven to nineteen and place them face down on the floor. Provide the students with two collections of counters. One collection should consist of bundles of ten counters, all of the same colour. The second collection should consist of single counters of assorted colours. Students take turns to select a card. They then collect a corresponding number of counters, using the bundles of ten and single counters. Encourage students to count on from the bundle of ten. This activity may be varied by extending the range of numbers or by using ten strips (made of ten dots on strips of card) instead of counters.

Why?

Developing an understanding of tens and ones will assist students in using strategies other than counting by ones to solve problems.

Where are they now?

Students do not see ten as a countable unit. The student's focus is on the individual items that form a unit of ten. In addition or subtraction tasks involving tens, students count forwards and backwards by ones.

Where to next?

Students see ten as a unit composed of ten ones and are able to use the unit to count. However, they are dependent on visual representations of ten, such as strips of card showing ten dots or using ten fingers.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1	Counts, orders, reads and represents two- and three-digit numbers
WMS1.1	Asks questions that could be explored using mathematics in relation to Stage 1 content
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Base ten: level 1.

How?

Popsticks

Make a base board by folding a piece of paper or cardboard in half to form two columns. Label the columns as “units” and “tens”. Construct a set of numeral cards for the range one to nine on white cards. These cards will be used to represent numerals in the “tens” column on the chart. Construct a second set of numeral cards in the range zero to nine on coloured card. These will be used to represent numerals in the “ones” column . Provide bundles of ten white popsticks and a pile of coloured popsticks. Shuffle the two decks of cards separately. Place the cards face down between the students. The students take turns to turn over a white card and a blue card to form a two-digit numeral and place the cards onto the chart. The students then read the numeral they have formed and collect a corresponding number of sticks, using the bundles of white popsticks and the coloured popsticks. Students then place the popsticks next to the numeral cards and allow others to verify that the number of popsticks used is correct.

Why?

Developing an understanding of tens and ones will assist students in using strategies other than counting by ones to solve problems.

Where are they now?

Students do not see ten as a countable unit. The students' focus is on the individual items that form a unit of ten.

Where to next?

Students see ten as a unit composed of ten ones and are able to use the unit of ten when counting, aided by visual representations of ten, such as ten strips.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1 Counts, orders, reads and represents two- and three-digit numbers

WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Backward number word sequence
Forward number word sequence
Numeral identification
Base ten strategies: intermediate concept of ten.

How?

Flip and see

Provide each student with a large collection of popsticks and a base board divided into a "tens" and a "ones" column. Place numeral cards in the range zero to nine face down on the floor. The students take turns to flip over two numeral cards and place one card in the tens column and one card in the ones column on their base board. Students then bundle popsticks into tens and place the correct number of bundles and units onto their base board to match the numeral cards. Discuss how many tens and ones were made.

Variations

- Students complete the above activity and then swap the numeral cards from the tens column to the units column and vice-versa. They then repeat the activity.
- Construct two sets of numeral cards in the range zero to nine. Flip over two numeral cards and ask the students to select identical numeral cards from the second set of cards. Ask students to place their cards in the tens and ones column so that they form the largest and the smallest number possible.
- Organise students into pairs and provide each pair of students with a set of numeral cards in the range zero to nine. The students shuffle the cards and place them face down on the floor. They then take turns to select two numeral cards. Using the two cards selected, each student forms the largest two-digit number possible. The two students then compare their numbers and the player with the larger number scores ten points. Continue playing until one player gains a score of one hundred.

Why?

Developing an understanding of tens and ones will assist students in using strategies other than counting by ones to solve problems.

Where are they now?

Students:

- are able to automatically represent a number on their fingers. They can usually produce the number word after a given number word in the range of one to twenty.
- are able to find the total of two groups of objects without touching the objects but need to start from "one".

Where to next?

Students:

- are able to count on from a given number to find the total of two groups
- automatically produce the number word after a given number within the range of one to thirty
- are able to count forwards to thirty and backwards from thirty or beyond.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1	Counts, orders, reads and represents two- and three-digit numbers
NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.1	Asks questions that could be explored using mathematics in relation to Stage 1 content
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems.

LFN reference

Forward number word sequence
Backward number word sequence
Counting on

How?

Bucket count on

Drop a small collection of blocks one by one, into a bucket. Ask students to count aloud as each block is added to the container. After dropping the blocks, show the students the contents of the bucket. Then hold the bucket above the eye level of the students. Ask the students to state how many blocks would be in the bucket if one more block was added. Repeat the question, changing the number of blocks to be added to two and three blocks. Encourage the students to count on from the number of blocks already in the bucket to find the total.

Variation

Ask the students to pretend there are a nominated number of blocks in the bucket. Drop additional blocks into the bucket. Students count on to find the total sum of the blocks in the bucket.

Why?

These activities encourage students to count on from a given number and assist in developing their knowledge of the forward sequence of number words.

Where are they now?

Students:

- can produce the number word that follows a given number word, in the range of one to twenty
- can say the backward number word sequence from "twenty" to "one"
- can say the number word just before a given number word, but may drop back to counting from "one" when doing so.

Where to next?

Students:

- are able to count on from a given number
- automatically recall number facts for ten
- automatically say the number word before or after a given number.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS1.1 Asks questions that could be explored using mathematics in relation to Stage 1 content.

LFN reference

Figurative counting
Base ten strategies
Quinary strategies

How?

Blocks on a bowl

Place a container, such as an empty ice-cream container, between a pair of students. Turn the container upside down and place five Unifix blocks on top. Instruct students to look away while their partner takes away some, or all, of the blocks from the top of the container and hides them under the container. The first student turns back to see how many blocks are left on top of the container. Using this information, the student determines how many blocks were placed under the container. The student may then lift the container to confirm the answer.

Variation

As students become competent with five blocks, ten and then twenty blocks could be used.

Why?

Providing students with opportunities to solve tasks involving hidden or screened items may encourage them to use "counting on" as a problem solving strategy.

Where are they now?

Students :

- are able to identify numerals in the range of one to twenty
- are competent in saying the forward number word sequence to twenty.

Where to next?

Students:

- are able to identify one- and two-digit numerals
- are able to say the forward number word sequence to 100.

Ensure squares on the 10 by 10 grid are large enough to easily accommodate the size of a counter.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1	Counts, orders, reads and represents two- and three-digit numbers
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday language and some mathematical language, actions, materials, diagrams and symbols.

LFN reference

Numeral identification

How?

Hundred chart

Display a large hundred chart. Ask students to identify, and explain to the group, any number patterns they can see on the hundred chart. After practice with the large hundred chart, give the students small hundred charts to work with. Cut the charts into strips so that numerals are grouped into tens. Students then sequence these numeral strips to recreate the hundred chart. Students then use these hundred charts to discover and record their own number patterns. Provide the students with calculators to confirm the number patterns.

Variations

- Provide students with a 10 by 10 grid to represent a hundred chart and fifteen counters, each displaying a different numeral in the range of one to one hundred. Students place the counters onto the grid in the correct numerical position. It may be necessary to provide a "key" numeral amongst the fifteen counters, for example the numeral 50.
- Cut groups of numerals on a hundred chart into a square formation. For example, cut the chart so that the numerals 1, 2, 11 and 12 are together. Alternatively, cut the hundred chart in a random pattern similar to a jigsaw design. Students then restore the hundred charts.
- Display a blank hundred chart on an overhead projector. Students call out numbers up to one hundred. The teacher, or a student, writes the nominated numerals onto the chart as they are named.
- Provide students with individual hundred charts. Blank out some of the numerals on the chart. Have the students write the missing numerals in the spaces.
- Allow opportunities for students to practise oral counting forwards and backwards as well as skip counting by twos, fives and tens.

Why?

These activities assist students in developing their knowledge of the relative size of numbers to 100.

Where are they now?

Students are able to complete addition tasks, but count from "one" to find the total, rather than counting on from the larger group.

Where to next?

Students use counting on as a strategy to solve addition problems.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.3	Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained

LFN reference

Counting on

How?

Add two dice

Construct a set of numeral cards in the range of two to twelve. Place them face up on a table, or on the floor. Taking turns, the students are to roll two dice and find the total. Encourage the students to count on from the larger number rolled. After adding the two dice the student takes a numeral card corresponding to the total. The game continues until all the cards have been taken. If a player rolls a number that has already been taken, the player's turn is forfeited.

Why?

Students need to be encouraged to use increasingly efficient strategies to solve addition tasks.

Where are they now?

Students are able to complete addition tasks, but count from "one" to find the total, rather than counting on from the larger group.

Where to next?

Students use counting on as a strategy to solve addition problems.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday language and some mathematical language, actions, materials, diagrams and symbols.

LFN reference

Counting on

(Add two dice)

Variations

- Use a variety of dice to extend the range of numbers. For example, construct a dodecahedron (12 faces) from plastic polygon shapes and attach numerals to each face of the construction. Modify the set of numeral cards to the appropriate range of numbers.
- Provide the students with three dice and with numeral cards for three to eighteen. This activity is then played like *Add two dice*. Students roll the three dice and find the total. This provides opportunities for introducing strategies other than counting by ones to solve addition tasks.
- Allow students to construct their own die and attach numerals of their choice. If large numbers are written on the first die, then modify the second die to display only the numerals 1, 2 and 3. A calculator may be used to confirm the additions.
- Another variation of the activity is achieved by instructing the students to write five numbers, within a nominated range, on a strip of paper. Students take turns to roll two dice and find the total. They then tell the group the total. As the totals are called, students cross off any corresponding numerals on their paper strip. The game continues until one student has crossed off all five numerals on his or her paper.

Why?

Students need to be encouraged to use increasingly efficient strategies to solve addition tasks.

Where are they now?

Students are able to solve addition problems up to ten or beyond, but they count from one to do so.

Where to next?

Students automatically recall addition number facts to ten.

Outcomes

This activity provides opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday language and some mathematical language, actions, materials, diagrams and symbols.

LFN reference

Figurative counting
Finger patterns

How?

Rabbits' ears

Instruct the students to make two fists and rest them on their heads, so that their hands are out of their direct line of sight. Ask the students to raise a given number of fingers on each hand and to add them together. Students may bring their hands down to confirm the answer.

Doubles

Instruct the students to use two hands to demonstrate double numbers from 1 to 5. For example, "Show me double four. How many altogether?" In this example the students would raise four fingers on each hand and call out the answer. Students may bring their hands down to count and confirm the total.

Doubles plus one

This activity is played in a similar way to *Doubles*. Instruct the students to raise their fingers for a nominated double combination and then add one more finger to find the total. Alternatively, play *Doubles minus one*. For this activity students raise their fingers to represent a nominated double and then subtract one finger to find the total.

Why?

Students use finger patterns to support their early understanding of numbers. Understanding that numbers are made up of other numbers allows students to move to strategies which don't involve counting by one. Using doubles and near doubles is a common early use of number facts.

Where are they now?

Students:

- instantly recognise die patterns to "six"
- are able to add two screened groups after being told the amount in each group, but count the total starting from "one".

Where to next?

Students:

- use the strategy of counting on from the larger group when adding two groups to complete addition tasks
- recall number facts to "ten".

Encourage students to predict which number will need to be to found to make a total of ten, before turning over the second card.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

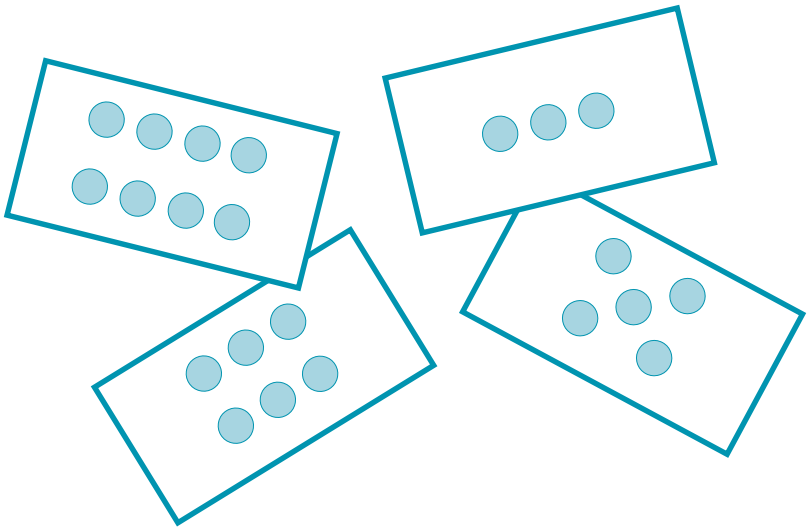
Counting on
Subitising

FIGURATIVE

How?

Dot pattern cards

Construct dot patterns for numbers one to nine, with two copies of card 5. Place the cards face down on a table between pairs of students. The students take turns to turn over two cards and add the two cards together. If the total is "ten" the student keeps the two cards. If the cards do not equal "ten" they are returned to the table. Encourage students to count on from the larger number.



Variations

- Use numeral cards instead of dot pattern cards.
- Use five numeral cards and five dot pattern cards.
- Use ten frame cards.
- Extend the total to 15 or 20.

Why?

Students need to develop a range of strategies to solve number problems. Strategies such as "counting on", "counting down to" and "counting up to" may be an efficient method of solving a problem. However this is dependent upon the task and the student knowing which is the most appropriate strategy.

Where are they now?

When adding two groups, the students count from "one" to find the total.

Where to next?

Students use the strategy of counting on from the larger group when adding two groups.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Counting on

How?

Posting counters

Provide the students with a container similar to a money box. Instruct the students to "post" a nominated number of counters through the slot in the container. Encourage the students to count each counter as it is dropped through the slot. Students then pause and state the total number of counters that are now hidden in the container. Instruct the students to post an additional number of counters into the container, counting on from the original number. Alternatively, ask students to pretend there are a specified number of counters in the container. Students count on as additional counters are posted through the slot.

Two-dice toss

Provide each pair of students with two dice and a pile of counters. Have the students take turns to throw the dice and add the total, counting on from the larger number. Students then take the corresponding number of counters from a central pile. The game continues until all the counters have been removed from the central pile.

Why?

Students need to develop strategies other than counting all items in groups, starting from one, when solving addition tasks. Counting on is a more efficient strategy.

Where are they now?

Students add two groups by counting from "one" rather than counting on from the larger group.

Where to next?

Students use the strategy of counting on from the larger of two groups to solve addition tasks.

Teachers may take this opportunity to model addition using counting on procedures.

Outcomes

These activities provide opportunities to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday language and some mathematical language, actions, materials, diagrams and symbols.

LFN reference

Counting on
Combining procedures

How?

Teddy tummies: addition

Prepare base cards using BLM on page 212. Provide each pair of students with ten counters and the appropriate base board. The students take turns to distribute the counters between the two teddies by placing the counters on the teddies' tummies. Pairs of students then discuss the number combinations formed with the ten counters. The students continue the activity, investigating the possible number combinations for the ten counters.

Variations

- The students record the number combinations for "ten" as they are discovered. Allow opportunities for the students to demonstrate their "discoveries" to the rest of the class.
- Substitute the "teddies" blackline master with BLM on page 213 displaying two trucks. Provide pairs of students with ten plastic teddies to complete the activity in a similar way to *Teddy tummies*.

Why?

Students need opportunities to use counting on as an addition strategy and to develop knowledge of the numbers that combine to make ten.

Where are they now?

Students demonstrate their understanding of the process of addition by joining two groups. They are able to find the total of the two groups by counting all of the items, starting from "one".

Where to next?

Students automatically recall number facts to 10.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday language and some mathematical language, actions, materials, diagrams and symbols.

LFN reference

Counting on
Partitioning and combining strategies

How?

Friends of ten

Construct two sets of numeral cards in the range of one to ten. For this activity it is necessary to attach string or shoelaces to the numeral cards so they can be worn around the students' necks. It is also more manageable if each set of cards is a different colour. Distribute one set of numeral cards to ten students. These students leave the room or turn away from the remaining students. Distribute the other set of numeral cards to the remaining students. Ask the students in the first group to return to the class (or turn around) and find a partner who is wearing a card which, when added to their own card, will equal ten.

Variations

- Increase the range of numbers on the numeral cards.
- Change the cards so that one set displays numerals and the other set displays dot patterns.

Why?

Knowing the basic number combinations that form ten allows students to use a range of strategies for addition, for example, knowing that 7+3 is the same as 8+2, and introduces the idea of compensation (one up, one down).

This activity develops the concept that addition may be an appropriate strategy for solving a subtraction problem.

Where are they now?

Students demonstrate their understanding of the process of addition by joining two groups. They are able to find the total of the two groups by counting all of the items, starting from "one".

Where to next?

Students automatically recall number facts to 10.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

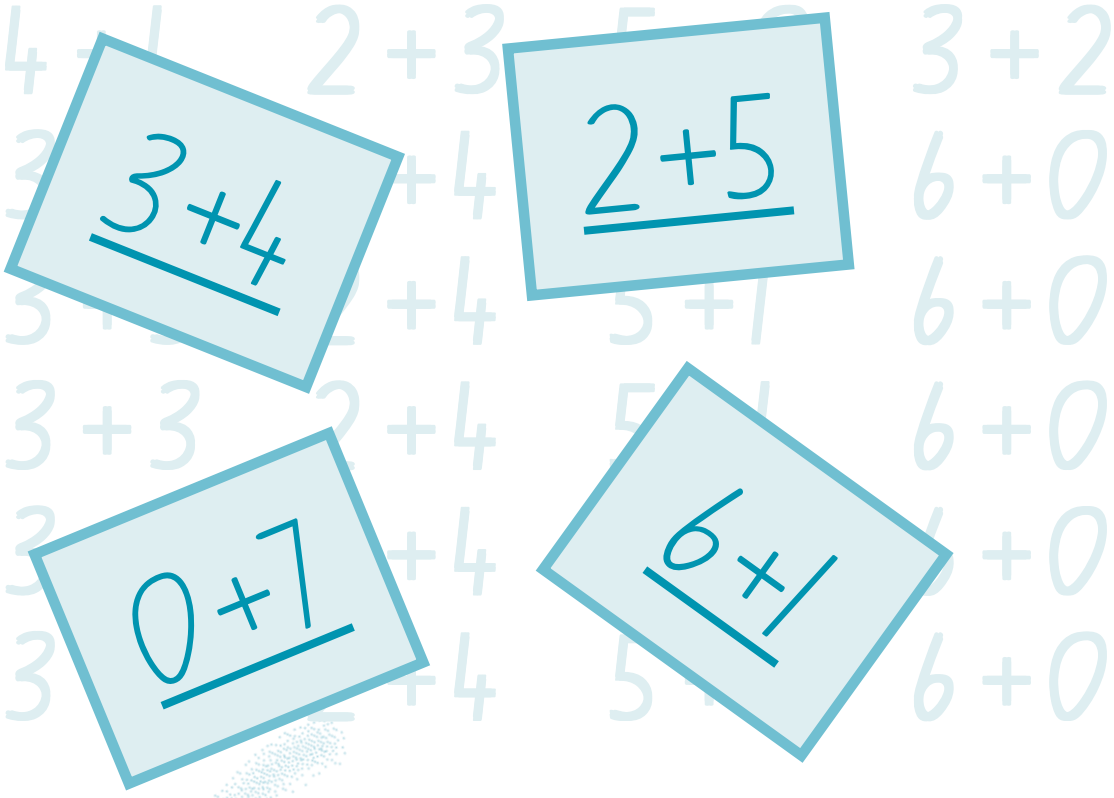
NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.3	Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained

LFN reference

Counting on
Partitioning and combining strategies

Fishing addition

Construct six sets of four cards, with each card in any one set displaying a basic addition fact for the same number in the range one to ten. Each set of cards should have a unique sum. Shuffle the cards and deal five cards to each player. Place the remainder of the cards face down on the floor. Instruct the students to look at their dealt cards and take turns to discard any pairs of cards that add up to the same total. After discarding any pairs of cards, the first player asks his or her partner for a card which equals a specific number. For example: "Pass me an eight". If the partner holds a card totalling the nominated number, it must be handed over. If the partner does not have the nominated card, the person who asked takes a card from the central pile. In both cases, if a pair of cards is formed, the student discards them. The winner is the player who discards all of his or her cards first.



Why?

Knowing basic number combinations that form ten allows students to use a range of strategies for addition, for example, knowing that 7+3 is the same as 8+2, and introduces the idea of compensation (one up, one down).

This activity develops the concept that addition may be an appropriate strategy for solving a subtraction problem.

Where are they now?

Students:

- can identify numerals from one to twenty
- are able to say the forward number word sequence to twenty
- confidently repeat the backward number word sequence from twenty.

Where to next?

Students:

- can identify numerals to 100 or beyond
- are able to say the forward number word sequence to 100 or beyond, confidently crossing into the next group of ten numbers.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2 Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems

WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Numeral identification: level 3

How?

Using technology

Provide the students with at least one calculator for each pair of students. Instruct the students to:

- enter a given number on the calculator
- enter on the calculator the number which is one before or one after a given number
- use the constant function on a calculator to add on from a given number.

Provide the students with a hundred chart on which they can record the number patterns.

Why?

This activity provides an opportunity for students to develop their knowledge of numbers up to and perhaps beyond 100.

Where are they now?

Students can solve addition tasks but count from one to find the total of two groups. Students are able to solve subtraction tasks when concrete material is available.

Where to next?

Students are able to complete addition and subtraction tasks using the strategies of "counting on" and "counting down from".

Allow students to discover that some numbers cannot be subtracted if they do not have enough counters to begin with.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.3	Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Combining and partitioning

How?

Ring that bell

Provide the students with a supply of counters. Ring a bell a number of times, for example, four times. Instruct the students to place the corresponding number of counters on their desk. Hold up a symbol card for addition or subtraction and then ring the bell again, for example twice. Have the students respond by observing the symbol card and adding or subtracting the correct number of counters. Students then state the total number of counters. Encourage the students to discuss their actions and how they arrived at their answers.

Variation

This may be used as a small-group or partner activity, with students rolling a die instead of ringing a bell.

Why?

Students need opportunities to use more efficient number strategies when completing addition and subtraction tasks.

Where are they now?

Students are able to find the total of two groups by counting from "one".

Where to next?

Students:

- use a counting on strategy to solve addition tasks
- count in a forward and backward sequence to 100 and beyond
- have automatic recall of addition and subtraction facts to ten.

Introduce the activity to the whole class before students complete the activity in small groups or with partners.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

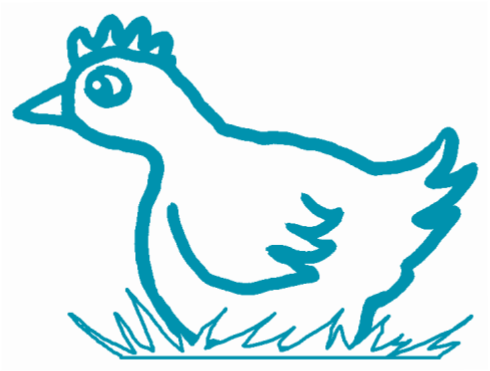
Figurative counting
Combining
Spatial patterns
Forward number word sequence
Backward number word sequence

How?

How many eggs?

Construct a grid using BLM on page 209 as well as one set of rule cards and a set of numeral cards for each group of students. Rule cards need to display either an addition or subtraction sign, followed by the numeral 1, 2 or 3. For example, a rule card might display: " + 3".

Instruct the students to place the correct number of counters on each column according to the numeral written at the top of the column. Students then take turns to turn over a rule card from the pile. Students follow the rule, to add or subtract counters from each column and determine the new total. As the students determine each total, they place a corresponding numeral card at the bottom of each column.



Variation

Modify the numerals at the top of each column.

Why?

This activity provides an opportunity for students to develop awareness of number patterns and to develop strategies for solving problems other than counting by ones.

Where are they now?

Students are able to solve addition tasks, but count from one to find the total of the two groups.

Where to next?

Students complete addition or missing addend tasks using the strategies of counting on or counting back.

This activity can be demonstrated by the teacher using an overhead projector and transparent counters.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Figurative counting

How?

Ten frames

Provide each student with a blank ten frame and ten counters. Ask students to place a specified number, less than ten, on their ten frame. Direct students to add another specified number of counters to the frame and to find the total.

Variations

- Complete subtraction tasks using the ten frames. For example, arrange nine counters on the ten frame. Ask students to determine how many counters would need to be taken away to leave six. Demonstrate the strategies of counting down to and counting down from a given number.
- Use two ten frames to work with numbers to twenty.

Why?

Ten frames provide students with a visual structure for a number. This encourages students to establish and work with visual images of numbers. Ten frames emphasise doubles and five as part of a number. For example, using a ten frame, 9 can be seen as 5+4, one less than double 5 or one more than double 4.

Where are they now?

Students:

- understand and demonstrate the meaning of subtraction by taking an object, or a group of objects, from a group of objects.
- are able to count in a backward number word sequence from twenty.

Where to next?

Students are able to use addition or to count down from a number to find the answer to a subtraction problem.

A full explanation of the strategies of *Counting down to* and *Counting down from* appear on the opposite page.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.2	Uses a range of mental strategies and informal recording methods for addition and subtraction involving one- and two-digit numbers
WMS1.3	Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

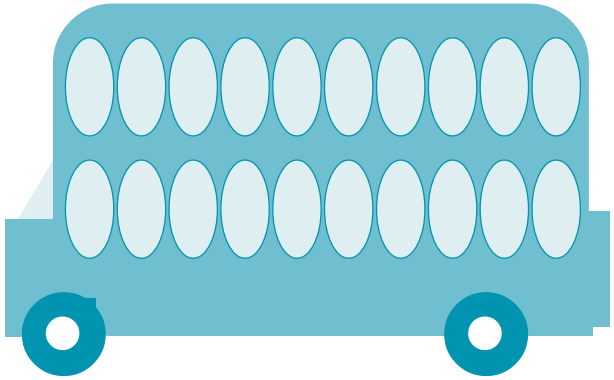
LFN reference

Figurative counting

How?

Subtraction teddies

Provide each student with twenty plastic teddies, a "double decker bus" baseboard (see BLM on page 214) and a strip of paper. Have the students place the twenty teddies on the bus baseboard. Instruct the students to take turns to roll a die and subtract the corresponding number of teddies from the collection of teddies on the bus. The student then records the number of remaining teddies on the strip of paper. The activity continues until one student reaches zero.



Counting down to

The student counts backwards from the larger number when solving subtraction problems where the problem involves a missing addend. For example, when solving 9 - ( ) = 6, the students would count backwards from nine knowing they are counting to the number six and say "eight, seven, six." Students typically hold up fingers as they count and recognise three as the answer.

Counting down from

The student counts backwards from the larger number when solving subtraction problems. For example, when solving 9 - 3, the student counts backwards from nine saying "eight, seven, six...six!"

Why?

This activity is designed to develop students' knowledge of subtraction facts. Automatic recall of addition and subtraction facts allows students to attend to other features when solving problems.

Where are they now?

Students count from "one" to solve tasks involving equal multiples.

Where to next?

Students are able to solve multiplication tasks by counting in multiples.

Encourage students to discuss how they reached their answer. Allow them to verbalise how they counted.

If the students are unable to skip count by threes, encourage rhythmic or stressed counting.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1	Counts, orders, reads and represents two- and three-digit numbers
NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

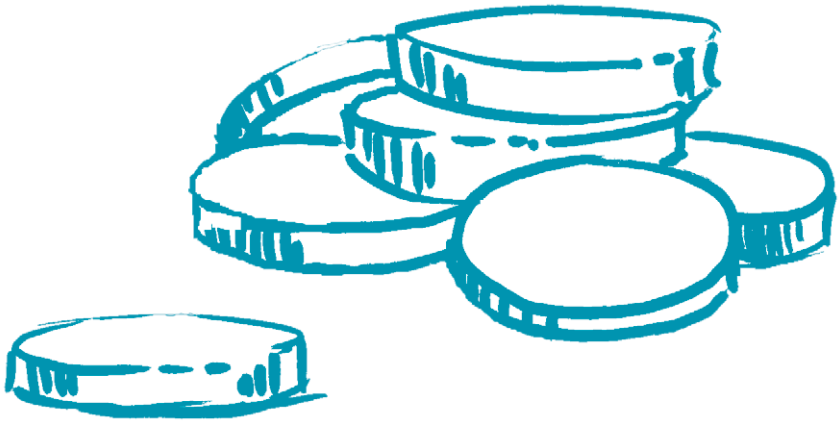
Early multiplication and division

How?

Arrays

Organise students into pairs. Provide the students with a collection of counters. Instruct one of the pair to make a simple array that is no larger than 5 by 5, with counters. The student then briefly shows the array to his or her partner before screening the counters with a sheet of cardboard.

The other student then attempts to construct the same array pattern with counters. The students should then compare the two arrays. Ask the students to find the total number of counters in the array.



Arrays: changing groups

Arrange nine students into three rows with three students in each row.

Pose the following question: "If we add another row of children, how many would there be altogether?"

Continue adding rows of students and encourage students to guess how many children there are altogether, prior to counting the students.

Variation

Change the number of students in each line.

Why?

Early multiplication and division strategies focus on the structure and use of groups of items. Students need to develop strategies where they see a group of items as one unit and no longer need to count each item within the group.

Where are they now?

Students need visible items to calculate the total number of items in a simple square array pattern.

Where to next?

Students are able to calculate the total number of items in a simple square array pattern without relying on visual representations.

Observe and question the students to determine how they are calculating the total in the array.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3 Uses a range of mental strategies and concrete materials for multiplication and division

WMS1.2 Uses objects, diagrams, imagery and technology to explore mathematical problems

WMS1.4 Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Early multiplication

How?

Guess my square

Organise students into pairs so that they are sitting opposite each other. Provide each student with an equal number of multi-link cubes or pattern tiles and a sheet of cardboard which will be used as a screen. Ask students to take turns to construct a simple array pattern using the material provided. Students should screen the array from their partner until the pattern is completed. The student then briefly shows the array to the partner before screening the array again. The other student constructs the same array pattern from memory. Instruct students to uncover and compare both array patterns. The students then calculate the total number of items in each array.

Variation

Arrange students in pairs. Ask the students to take turns to make an array which is hidden from their partner's view. The student then describes the array to the partner by stating the number of counters in each row and the number of rows. The partner then attempts to make the array. He or she then determines the total number of counters in the array.

Why

The formation of arrays is important in developing strategies of coordinating groups. This coordination of groups can lead to students developing abstract concepts to solve multiplication and division problems.

Where are they now?

The students determine the total number of items within a specified number of groups by counting the total by "ones".

Where to next?

Students use the strategies of rhythmic counting and skip counting to find the total number of items within a specified number of groups.

Literature link

Many traditional folk and fairy tales have a theme based on three, such as The Three Little Pigs and Goldilocks and the Three Bears. Grouping based on these tales can be used in activities.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.3	Describes mathematical situations and methods using everyday and some mathematical language, actions, materials, diagrams and symbols
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

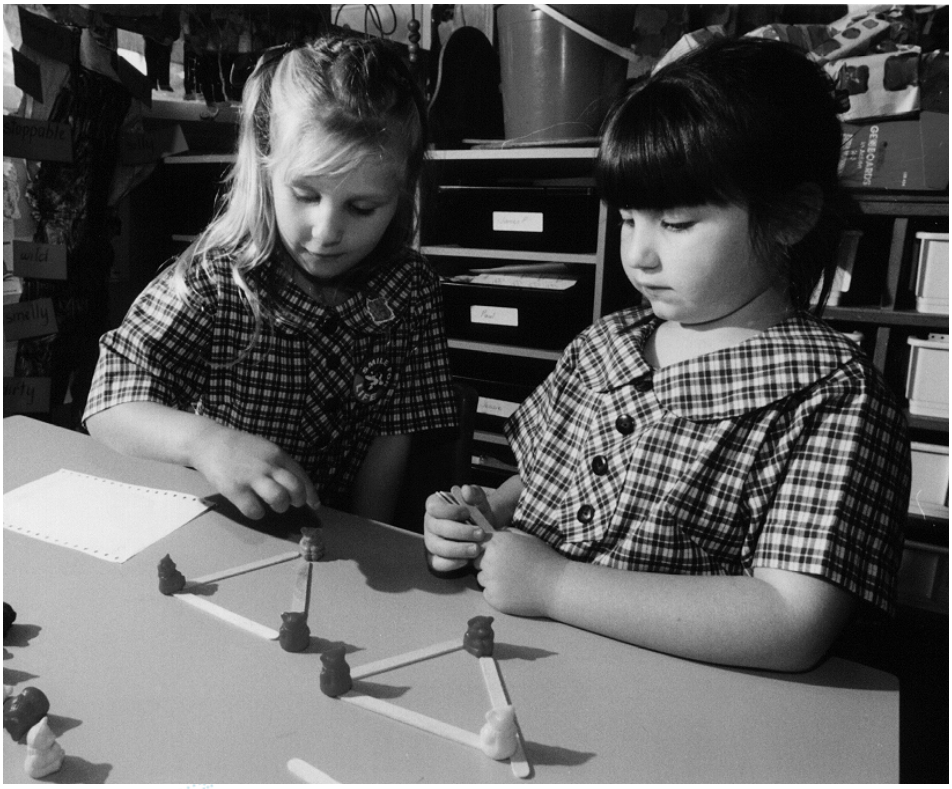
LFN reference

Early multiplication and division

How?

Triangle teddies

Provide the students with a collection of popsticks and a collection of plastic teddies. Instruct the students to make a triangle using three sticks. Ask the students to then place a teddy on each of the corners of the triangle. The students then count and record the number of teddies on the triangle. Have the students repeat the process for a second triangle. The students record the total number of teddies placed on the two triangles. Have the students continue to form additional triangles and record the total number of teddies on the triangles.



Why?

Students need to be able to recall the counting sequences for nominated multiples. Multiplication and division strategies will be limited by students' knowledge of these sequences.

Where are they now?

The students determine the total number of items within a specified number of groups by counting the total by "ones".

Where to next?

Students use the strategies of rhythmic counting and skip counting to find the total number of items within a specified number of groups.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.1	Counts, orders, reads and represents two- and three-digit numbers
NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Early multiplication and division

How?

Units for two

Collect or draw items commonly found in pairs, such as eyes, shoes, socks, or legs. Model the method of counting the items, using rhythmic counting, to the students. Model methods for keeping track of the number of groups as well as the total number of items. Allow opportunities for the students to practise the modelled methods. Demonstrate how the total number of items in a specified number of "groups of two" can be found by counting the first number in each pair silently and voicing the second number. Allow opportunities for the students to practise this counting method.

Handprints

Make a handprint to represent a group of five. Repeat printing the handprints across a strip of paper. Ask the students to count the number of hands and to find the total number of fingers using rhythmic or skip counting methods.

Variation

Construct a square using four popsticks to represent groups of four.

Why?

Students need to be able to recall the counting sequences for nominated multiples. Multiplication and division strategies will be limited by students' knowledge of these sequences.

Where are they now?

Students are able to form equal groups of items but find the total number of items in the groups by counting by ones.

Where to next?

Students are able to use rhythmic and skip counting strategies to find the total of equal groups.

Write the instruction cards in two ways, such as four rows of 5 and five rows of 4.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

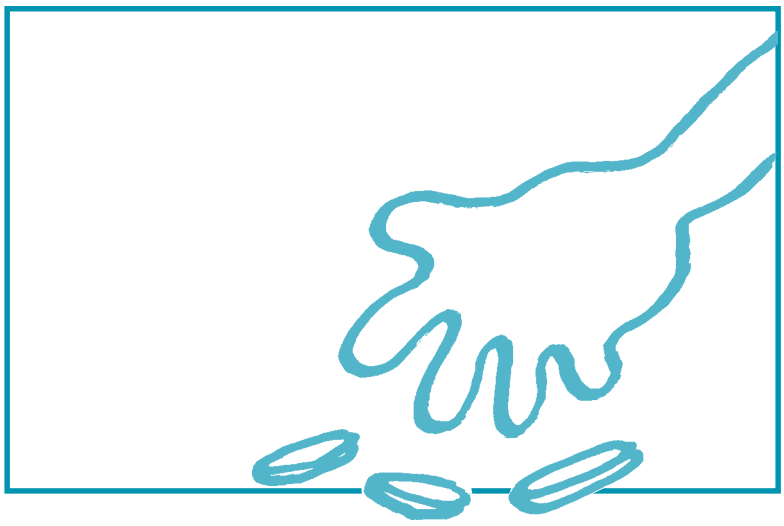
LFN reference

Early multiplication and division: level 2

How?

Turning arrays

Provide each student with a small sheet of cardboard and a supply of counters. Instruct students to form arrays by placing the counters onto the cardboard following instructions, such as "make three rows of five counters". Students then turn the card 90° to show a new array of five rows of three. Discuss with the students the number of rows, the number of counters in each row and the total number of counters for each array pattern.



Variation

Allow the students to form arrays using potato prints, shape prints, thumb prints or adhesive stickers. Provide students with instruction cards for making the arrays.

Why

The formation of arrays is important in developing concepts of groups and coordinating groups. This coordination of groups can lead to abstract concepts of multiplication and division.

Where are they now?

Students are able to form items into equal groups but count by ones to find the total.

Where to next?

Students are able to use the strategies of rhythmic and skip counting to find the total of equal groups.

BLMs could be covered with contact and felt pens used to circle the groups. They can then be erased for later use by others in the class.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Early multiplication and division: stages 1, 2

How?

Calculating groups

Provide students with a stencil displaying a large number of items. Instruct students to draw rings around groups of items on the stencil. For example: "Put a ring around groups of five items." Ask the students to determine how many groups were made and to use rhythmic counting or skip counting to find the total. Ask the students to determine the number of single items remaining on the sheet.

What's in a square?

Construct multiple copies of the grid on BLM on pages 210 and 211 and accompanying picture cards displaying groups of items. Each pair of students will need a copy of the grid, a deck of picture cards and counters of two different colours. Shuffle the picture cards and place them face down before the students. The first player takes a card from the top of the pile and places a counter on the corresponding square on the grid. For example, if a card displaying one item is drawn, the student places his or her counter on the "one group of 1" square at the top left-hand corner of the grid. Players continue to take turns to turn over cards and mark them on the grid. The winner is the first player to make a line of three counters horizontally, vertically or diagonally.

To extend this activity introduce numeral cards which indicate the total number of items on each picture card. After the students place a picture card onto the grid, instruct them to determine the total of the groups and place a corresponding numeral card on top of the picture card.

Why?

Early multiplication and division strategies focus on the structure and use of groups of items. Students need to develop the concept of seeing a group of items as one "unit" and no longer relying on counting each item within the group.

Where are they now?

Students are able to form items into equal groups. They calculate the total number of items in the groups by counting by ones.

Where to next?

Students are able to use the strategies of rhythmic and skip counting to find the total of equal groups.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Early multiplication and division: stage 2

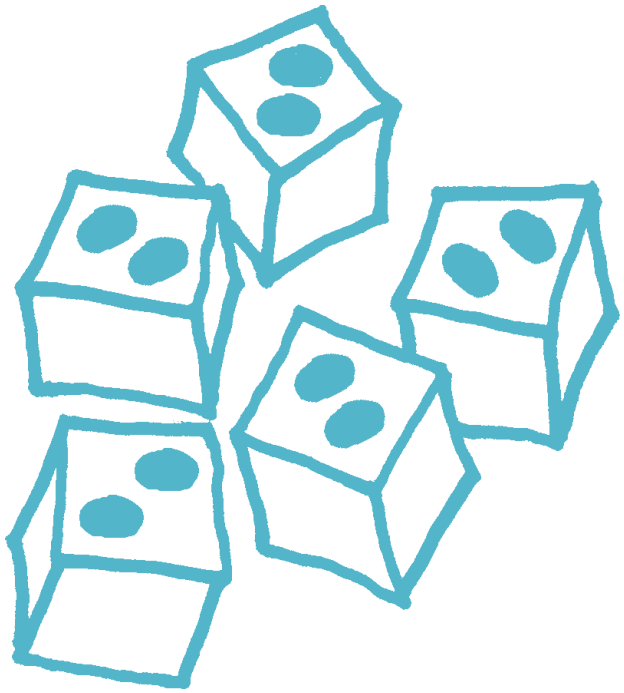
How?

Groupies

Provide the students with a collection of items such as counters, marbles or plastic teddies. Direct the students to form a specified number of equal groups using the items, for example, "Make four groups of three counters". Students use rhythmic or skip counting to find the total number of items in the specified groups.

Variation

Use dot patterns on dice to represent groups, for example, five groups of two.



Why?

Early multiplication and division strategies focus on the structure and use of groups of items. Students need to develop the concept of seeing a group of items as one "unit" and no longer relying on counting each item within the group.

Where are they now?

Students are able to form items into equal groups. They calculate the total number of items in the groups by counting by ones.

Where to next?

Students are able to use the strategies of rhythmic and skip counting to find the total of equal groups.

Outcomes

These activities provide opportunities for students to demonstrate progress towards the following outcomes: A student

NS1.3	Uses a range of mental strategies and concrete materials for multiplication and division
PAS1.1	Creates, represents and continues a variety of number patterns, supplies missing elements in a pattern and builds number relationships
WMS1.2	Uses objects, diagrams, imagery and technology to explore mathematical problems
WMS1.4	Supports conclusions by explaining or demonstrating how answers were obtained.

LFN reference

Early multiplication and division: stage 2

Dice and grid game

Construct a 6x6 grid for each player. Write the numerals 1 to 36, in sequence, onto the grid, placing one numeral in each square. Each player will also need six sets of dot pattern cards representing the numbers one to six, a numeral die and a dot pattern die.

In this activity, the "numeral die" is used to indicate the number of equal groups, and the "dot pattern die" indicates the number of items in each group.

Instruct the students to roll the two dice and state the size of each group and the number of equal groups. When the student has stated the size and number of equal groups, ask the student to collect the correct number of dot pattern cards to represent the groups. For example, if the student wishes to make five groups of three, he or she would then select five cards showing three dots on each card.

Students then find the total number of dots by rhythmic or skip counting, and place a counter on the corresponding numeral on the grid. In the above example, the student would place the counter on the numeral 15. The winner is the first to have three counters in a row, horizontally, vertically or diagonally.

Variation

Allow students to use a calculator to determine the total number of dots in the groups. Observe whether the student uses repeated addition or multiplication with the calculator.

Why?

Early multiplication and division strategies focus on the structure and use of groups of items. Students need to develop the concept of seeing a group of items as one "unit" and no longer relying on counting each item within the group.

Assessment tasks

Task	Student response	Assessment
T :"If I have 6 pencils and I get another 3, how many do I have altogether?"	S: Correctly adds the two groups without the use of concrete material.	Did the students count from one, or did they count on from 6?
T: Displays a set of eight counters. "Here are eight counters." Screens the counters. The teacher then displays a set of 5 counters. "There are five counters here". Screens the second group. "How many counters are there altogether?"	S: Demonstrates addition by counting on from the larger number.	Did the student: • count from one? • count on from eight? • know the number fact automatically? If the student was unsuccessful in adding the two groups, the teacher should unscreen the second group and pose the question again. This may encourage the student to count on.
T: "If I have 8 lollies and I eat 3, how many are left?"	S: Completes the subtraction task without the use of concrete materials.	Which strategy did the student use to determine the answer?
T: Displays and then screens a collection of 15 counters. "I'm taking out 6. How many are left?" Displays the six removed counters.	S: Completes the subtraction task without having to see or feel the counters.	Did the student • count on from 6? • attempt to count down from 15? • attempt to count down to 6?
T: Displays numeral cards in the range from 1 to 100, and asks the student to name the numeral being displayed.	S: Correctly names each numeral as the cards are displayed.	Is the student able to name numerals in the range: 1-10? 1-20? 1-100?
T: "Count forwards starting at 55" (and stop at 64).	S: Correctly says the forward number word sequence from 55 to 64.	Does the student: • know the forward number word sequence? • count fluently from one decade to the next? (e.g. 59, 60)

Assessment tasks

Task	Student response	Assessment
T: "Count backwards starting at 93" (and stop at 87).	S: Correctly says the backward number word sequence from 93 to 87.	Does the student: • know the backward number word sequence? • count fluently from one decade to the next? (e.g. 90, 89)
T: "Name the numeral which comes after " (numbers in the range 1 to 100, e.g. the number after 53)	S: Correctly says the numeral which comes after the numerals given by the teacher.	Did the student drop back and count up to find the next number?
T: "Name the numeral which comes before" (a given numeral in the range from 1 to 100, e.g. the number before 53 is 52)	S: Correctly says the numeral which comes before the numerals given by the teacher.	Does the student know the backward number word sequence from 100 to 1? Did the student drop back to a lower decade to determine the answer?
T: Briefly displays 7 counters before screening them. "Here are seven counters. I'm taking some of them out and there are 2 left. How many did I take out?"	S: Determines the missing addend without seeing or feeling the counters.	Does the student model and count the concealed items? Did the student count down from 7 or count down to 2? Did the student count from one, three times?
T: Displays a set of numeral fact cards (addition facts to ten.) "What does this say?" "Can you work it out?" "How did you do it?"	S: Automatically recalls number facts or uses an efficient strategy to solve the task.	Did the student automatically provide the answer to the number fact? If not, what strategy did the student use? Did the student: • count from one? • count on from the larger number?

Three-minute lesson breakers

- Oral counting using the hundred chart.

- Think of a "secret number" and provide the students with clues for guessing the "secret number". Include simple and complex clues.

  For example:

  " The secret number is two more than three."

  " The secret number is more than twelve but less than nineteen."

  " The secret number is an odd number between 10 and 20."

- Provide opportunities for the class to practise oral counting from a number other than one. For example, instruct the students to count from 36 to 50.

- Using one week on a calendar as a visual aid, pose such problems as:

  "Count the number of breakfasts or total meals a person would eat in one week."

  "Count the number of days already spent at school this week and the number of days left until the weekend."