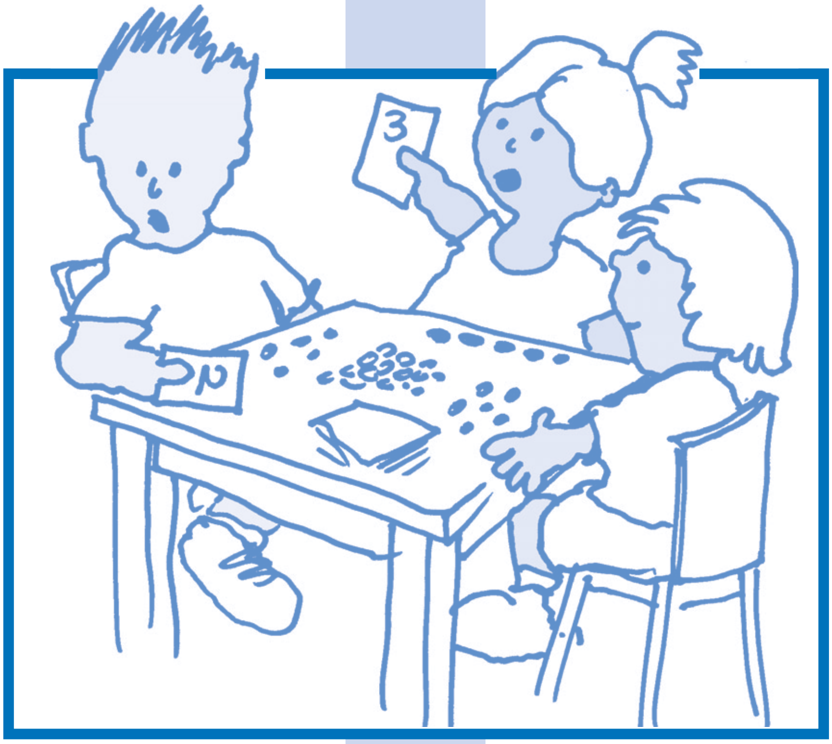
Developing efficient numeracy strategies



Stage1

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Foreword

Laying strong foundations in students' early numeracy learning is critical. As Napoleon noted, the advancement and perfection of mathematics are intimately connected with the prosperity of the State.

Students' methods for solving number problems can be diverse, even in the early years of school. Counting is a surprisingly complex and powerful procedure. The order in which the number words must occur, the one-to-one relationship which the number words have with items being counted and the realisation that the last number in a count tells us how many objects are in a set, are all part of counting.

Developing efficient numeracy strategies – Stage 1: Number provides clear directions for supporting student learning in number. The teaching activities within this book build upon the students' current methods of solving arithmetical problems. The focus for teaching changes from looking solely at students' answers to the strategies which each student uses to solve the problem.

The three questions:

What do my students currently know?

What do I want them to know? and

How will I help them to learn this?

are fundamental to teaching and are evident in each section of this book. This document is a practical resource to support the purposeful teaching of number strategies. It enables teachers to manage the teaching process so that it addresses the leading edge of each student's current knowledge.

I commend this document to you as a resource that will support the day-to-day teaching of number.

Robert Randall
Director, Professional Support and Curriculum

Contents

About this book

Numbers surround us each day of our lives. Young children encounter numbers in play, in nursery rhymes and as a recurring theme in conversations, e.g. "How old are you?" Yet counting as a procedure is far more complex than learning a telephone number or the alphabet. The process of counting develops from initial attempts to string together a sequence of number words to using counting in sophisticated ways to solve addition, subtraction, multiplication and division problems.

Developing a powerful and flexible understanding of how numbers are used is one of the goals of early mathematics learning. *Developing efficient numeracy strategies, Stage 1* provides teachers with a resource for programming activities in number to achieve this goal.

Many of the activities outlined in this book have been developed and trialled in the early numeracy project, *Count me in too*. This project used a learning framework in number, initially developed by Professor Bob Wright, to support observations of children's strategies for solving arithmetical problems.

All of the activities in this book have been designed to build upon students' current methods of solving arithmetical problems. The activities support and encourage students in using increasingly efficient strategies when completing arithmetical tasks.

The organisation of the material in this book emphasises both direction and purpose in the teaching of mathematics. The sections are sequenced to reflect development from emergent understandings of number through to efficient use of counting on strategies. Similarly, within each section the activities have been arranged to answer the fundamental questions of teaching mathematics:

What do my students currently know?

What do I want them to know?

How will I help them to learn this?

How to use this book

The layout of *Developing efficient numeracy strategies* is based on four central groups of early arithmetical strategies: emergent counting, perceptual counting, figurative counting and counting on. An overview of the key features of each of these groups of strategies is provided at the start of each section.

The main groups of strategies which students use can be organised into a progression. This progression of strategies is described within this book by four stages.

1. **Emergent counting stage**
   The student knows some number words but cannot count visible items. The student either does not know the correct sequence of number words or cannot coordinate the words with items.

2. **Perceptual counting stage**
   The student can count perceived items but not those in concealed collections. Perceptual counting includes seeing, hearing or feeling items.

3. **Figurative counting stage**
   The student can count concealed items but counts from one rather than counting on. Has a "figurative" notion of numbers and does not need to count perceived items, but counts from one to construct a number in additive situations.

4. **Counting on stage**
   The student can use advanced count-by-one strategies. Counts on rather than counting from "one", to solve addition tasks or tasks involving a missing addend.

These stages of number development provide the titles of each section.

Developing efficient numeracy strategies is organised into four main sections which mirror the above groups of strategies.

Each section contains:

- *Where are they now?*
- *Where to next?*
- *How?*
- *Why?*
- *Syllabus references.*

Where are they now?

Describes the types of approaches which students may use in attempting to solve problems.

Where to next?

Provides direction for teachers in determining where students are headed. It makes explicit the next level of sophistication in students' solutions.

How?

Outlines activities designed to assist students' arithmetical development. These activities are not sequenced within each section. You are free to modify the activities to suit the needs of your students.

Why?

Provides the purpose of the activities.

You will need initially to assess your students' current problem-solving strategies and counting skills. This assessment will support you in programming appropriate teaching activities.

Each section of this book is introduced by *The nature of the learner* and *Teaching considerations.*

The nature of the learner provides a summary of strategies which students typically demonstrate at each stage of arithmetical development.

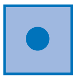
Short, practical assessment tasks are included at the end of each section. These tasks provide ideas for assessing your students' development and will enable you to determine if students have progressed to the next stage.

Other key features include *Teaching points* and *Three–minute lesson breakers.* These are identified by the following icons:

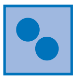
The *Teaching points* will help you organise the activities or provide further detail as to what to look for in students' responses.

The *Three–minute lesson breakers* are short whole-class activities requiring little or no equipment. They can be used to consolidate and practise the skills at each stage.

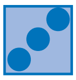
Icons are used throughout the book to indicate whether the activities are appropriate for individuals, partners, small groups or the whole class.



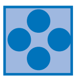
individuals



partners



small groups



whole class.

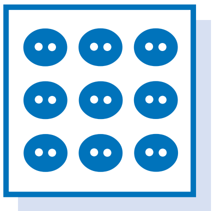
Introduction

Strategies for solving arithmetical problems

In becoming effective users of mathematics, students develop and use a range of methods of solving problems. These strategies tend to become more sophisticated as better ways of determining the answer are developed.

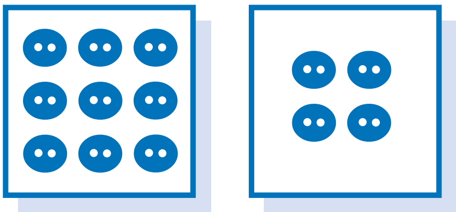
The range of strategies which students use to solve problems in mathematics is best described through an example.

Teacher: *How many buttons?*



Children learn the forward sequence of number words initially in the same way as they learn the alphabet, as a continuous string. To find the answer to this question they need to know the correct sequence of number words, to match the count to the objects and to recognise that the last number stated signifies the total.

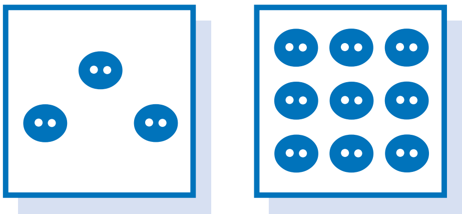
Teacher: *Now I have added some more buttons. How many altogether?*



A child who counts them all but starts again from one is using a less sophisticated strategy than a child who counts on from nine. To be able to count on from nine, children need to be able to cope with the forward sequence of number words. If you ask children what number comes after nine, they will often initially count from one to find the answer. In developing the ability to count on, students need to know the sequence of number words well enough to continue counting from any number.

Typically, students move through developing knowledge of the sequence of number words, to combining and counting all the objects they can see, to counting on and eventually to using addition facts.

Teacher: *If I have 3 buttons and I add 9 more, how many altogether?*



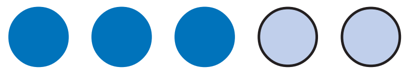
When approaching addition questions using a counting on strategy, a more sophisticated student will always count on from the larger number. Similarly, in answering 3 + 9, many students will often "bridge to ten", saying: "1 plus 9 is ten and I have 2 more, making 12".

The purpose of this example is to demonstrate the range of strategies students use and to emphasise the need to help them to develop sophisticated strategies. One of the difficulties with inefficient strategies is that, although they are slower, they still work. A student asked to find 8 + 3 can count out 8 then count out 3, and finally count all the objects to obtain an answer. If this strategy persists, the amount of mental effort needed to obtain the answer makes it difficult to achieve further learning.

It is important to learn to use addition facts automatically, as it allows the student to attend to other features of problems. Basic strategies can persist even after students develop more sophisticated approaches. Competent adults will occasionally revert to using their fingers to count on at times, because this strategy achieves the correct answer and doesn't require as much thinking as using addition facts.

As well as developing efficient "counting strategies" students need knowledge of the place value of numbers. This is aided by a growing understanding of ten as a unit. Many of the processes needed in addition and subtraction require students to "see" the ten in such numbers as 24. Understanding 24 as two tens and a four is essential in using the place value structure of numbers in addition and subtraction. Teaching activities, such as the Trading Game, emphasise the nature of tens and hundreds in numerals. As students trade up or trade down, they build the idea that each ten is a composite unit made up of ten ones. This same idea of forming a unit made up of smaller pieces is also of fundamental importance in measurement and in multiplication.

The notion of units within units is supported by such processes as combining and partitioning. Combining refers to bringing parts together; partitioning refers to separating the parts while maintaining a sense of the original number. It emphasises part-whole (sometimes called part-part-whole) number relations. That is, students see both the parts and the whole.

Can you see the 3, the 2 and the 5? 

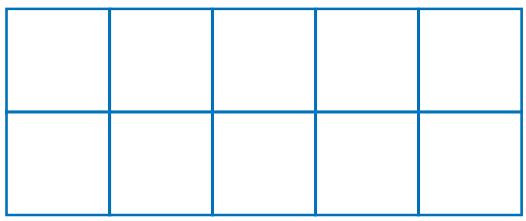
Interpreting number in terms of part-whole relationships makes it possible for children to think about a number as being composed of other numbers.

We often recognise the number associated with a particular pattern straight away, even before we have had time to "count" the items. This normally applies to small numbers of items.

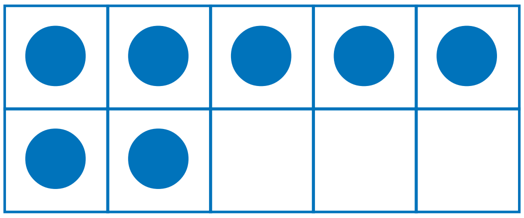
The process of immediately recognising how many items are in a small group is called subitising. This name comes from the Italian word *subito* which means "immediately" or "right now". When playing a game with dice we normally recognise the number of dots immediately.

The process of subitising can also be used with seeing parts in the whole. If you look at the dot pattern for five you become aware of seeing it also as four and one or three and two. Developing a rich knowledge of numbers such as five helps children to understand how each number is made up of other numbers.

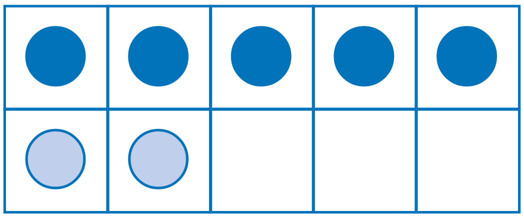
To assist students in forming a clear understanding of the base ten structure of numbers we can use organisers such as ten frames.



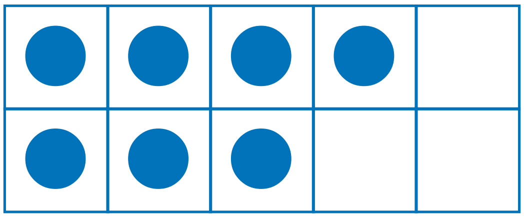
When these are used in conjunction with counters, students can model combining and partitioning in a structured way. Three counters and four counters can be combined to show the total of seven counters.



The ten frame also provides a visual link between seven and ten. Students can see that three more counters are needed to make ten. That is, the "negative image" is visible as well as the "positive image". It is also possible to use ten frames to explore partitioning numbers. If you ask students what they see when they look at the seven counters in the ten frame some will notice that seven is made up of five and two.



Indeed all the different number combinations or partitions of seven can be explored from the ten frame. If the ten frame is filled from the left it can be used to build students' knowledge of doubles and near doubles. For example, seven can be seen as one more than six (double three).



This knowledge of number combinations is very useful with addition and subtraction questions. To add 27 and 5 we often break the five into 3 and 2 to find the answer. This process of breaking numbers into parts is noticeable in mental calculations.

Just as ten is essential to our understanding of operations on numbers, five often acts as a base for mental calculation. This is due to students' early use of finger strategies with arithmetic. The ten frame is organised as five squares and five squares, which replicates the first organised material which students use, namely, two hands with five fingers on each.

An understanding of the framework of strategies which students use in early arithmetic enables teaching decisions to be based on knowledge of children's understanding of mathematics. The components of this framework are interrelated and interdependent. As an introduction to the nature of the framework, however, each component will be presented separately.